# A Statistical Inquiry into the Plausibility of Recursive Utility

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# Motivation

- There has been a revival of interest in consumption based asset pricing

  ▷ Much of the new literature uses recursive utility

  ▷ Some recent papers
    - ◇ Bansal and Yaron (2004)
    - ◇ Hansen, Heaton, and Li (2005)
    - ◇ Kiku (2006)
    - ◇ Eraker (2006)
    - ◇ Ai (2006)

- Natural question: Is recursive utility plausible?

# Statistical Methods

- Obvious approach is to specify consumption dynamics and fit the model implied by recursive utility to maco data
  - ▷ Bayesian methods are natural because macro data are sparse
  - ▷ Some recent papers:
    - ◇ Del Negro and Schorfheide (2004)
    - ◇ Gallant and McCulloch (2005)
    - ◇ Fernandez-Villaverde and Rubio-Ramirez (2004)
- Excess baggage: These methods require a driving process, a solution method, a likelihood synthesis, and rely on consumption data.
- We use a little known Bayesian method that relies on none of the above.

# Our Approach

- Pricing kernel: $\theta = (\theta_1, \ldots, \theta_{n+1})$
  - ▷ Extracted nonparametrically from a panel of returns $R_{it}$
- Hierarchical likelihood: $\mathcal{L}(\theta, \eta) = \mathcal{L}(\theta) \prod_{t=1}^{n} f(\theta_{t+1} | \theta_t, \ldots, \theta_1, \eta)$
  - ▷ $\mathcal{L}(\theta)$ is a function of returns derived from the Euler equation $1 = \mathcal{E}_t \theta_{t+1} R_{it}$ by a conditioning argument.
  - ▷ $f(\cdot|\cdot, \eta)$ is the seminonparametric transition density of the pricing kernel.
- Composite prior: $p(\theta, \eta) = p(\eta) p_T(\theta, \eta)$
  - ▷ $p(\eta)$ prior on hyperparameter $\eta$ : tight, intermediate, loose
  - ▷ $p_T(\theta, \eta)$ imposes technical conditions
- Inference:
  - ▷ MCMC posterior probabilities $P(A) = \int \int I_A(\theta, \eta) P(d\theta, d\eta)$

# Implementation

- Data are the twenty-five Fama-French portfolios and Treasury debt: monthly 1959–2004 and annual 1930–2004.

- Substantive prior $p(\eta)$ and law of motion $p(\cdot|\cdot,\eta)$ suggested by a Bansal-Yaron economy.

- Obtain the posterior distribution $P(\theta,\eta)$ of the pricing kernel $\theta$ and the parameter $\eta$ of its law of motion.

- Posterior probabilities computed: "Is Bansal-Yaron economy?", "Is recursive utility?", "Is external habit utility?".

- Events "Is utility?" are expressed terms of $R^2$ of regressions of the pricing kernel on variables measured with error.

## Findings

- Bansal-Yaron consumption dynamics mildly misspecified.
- Bansal-Yaron cash flow dynamics more seriously misspecified.
- External habit utility is implausible.
- Recursive utility is plausible.

# Outline

- Implications of recursive utility
- Data
- Law of motion of pricing kernel $f(\theta_t|\theta_{t-1},\ldots,\theta_1,\eta)$
- Prior $p(\eta)$ on hyperparameter $\eta$
- Likelihood for observables $\mathcal{L}(\theta)$
- Implementation of $\mathcal{L}(\theta)$
- Technical prior $p_T(\theta,\eta)$
- Empirical results

# Consumption Endowment and Cash Flows

$C_t$ consumption endowment

$P_{ct}$ price of an asset that pays the consumption endowment

$R_{ct} = (P_{ct} + C_t)/P_{c,t-1}$ gross return on consumption endowment

$D_{st}$ cash flow $S$

$P_{st}$ price of cash flow $S$

$R_{st} = (P_{st} + D_{st})/P_{s,t-1}$ gross return on cash flow S

Prices are real

# Recursive Utility

Marginal rate of substitution

$$M_{t,t+1} = \delta^{\beta} \left( \frac{C_{t+1}}{C_t} \right)^{-(\beta/\psi)} (R_{c,t+1})^{(\beta-1)},$$

$\delta$ time preference parameter
$\gamma$ coefficient of risk aversion
$\psi$ elasticity of intertemporal substitution
$\beta = \frac{1-\gamma}{1-1/\psi}$

Log-linear regression of the marginal rate of substitution on consumption growth and the return to consumption has an $R^2$ of one.

# In General

Euler equation

$$1 = \mathcal{E}_t \left( \theta_{t+1} R_{t,t+1} \right)$$

Price of Consumption Endowment

$$P_{ct} = C_0 \left( \prod_{k=1}^{t} \frac{C_k}{C_{k-1}} \right) \sum_{j=1}^{\infty} \mathcal{E}_t \prod_{k=1}^{j} \left( \frac{C_{t+k}}{C_{t+k-1}} \theta_{t+k} \right)$$

Satisfied by any pricing kernel $\theta_{t+1}$ including $M_{t,t+1}$ given by recursive utility

# Data: Two Data Sets

- First: 551 monthly observations from February 1959 through December, 2004
  - ▷ real returns including dividends on 24 Fama-French (1993) portfolios.
  - ▷ real returns on U.S. Treasury debt of ten year, one year, and thirty day maturities
  - ▷ real returns including dividends on the aggregate stock market
  - ▷ real, per-capita, consumption expenditure and labor income growth
- Second: The second is 75 annual observations from 1930 through 2004 on the same variables except U.S. Treasury debt of ten and one year maturities.
- Exclusion of one Fama-French portfolio and debt due to missing values.

# Law of motion of pricing kernel $f(\theta_t|\theta_{t-1},\ldots,\theta_1,\eta)$

- Fit SNP expansion to simulated data from a calibrated Bansal-Yaron economy.

- Use BIC to determine the truncation point.

- The form of the fitted density of $\theta_t = M_{t-1,t}$ is

$$
\begin{aligned}
f(\theta_t|\theta_{t-1},\ldots,\theta_1,\eta) &= f(y_t|y_{t-1},\ldots,y_1,\eta)/\exp(y_t) \\
y_t &= \log(\theta_t) \\
f(y_t|y_{t-1},\ldots,y_1,\eta) &= \mathcal{P}^2(y_t|a)\, n(y_t \,|\, \mu_{t-1},\sigma^2_{t-1}) \\
\eta &= (a_1,a_2,a_3,a_4,b_0,b_1,r_0,r_1,r_2) \\
\mathcal{P}(y_t|a) &= 1 + a_1 y_t + a_2 y_t^2 + a_3 y_t^3 + a_4 y_t^4 \\
\mu_{t-t} &= b_0 + b_1 y_{t-1} \\
\sigma^2_{t-1} &= r_0^2 + r_1^2\sigma^2_{t-2} + r_2^2(y_{t-1}-\mu_{t-2})^2
\end{aligned}
$$

# Prior $p(\eta)$ on Hyperparameter $\eta$

- Three versions using location and scale from fit to simulated data from a calibrated Bansal-Yaron economy.
  - ▷ Tight: Scale divided by ten; binds.
  - ▷ Intermediate: Scale as estimated; informative.
  - ▷ Loose: Scale multiplied by ten; uninformative.
- Intermediate scale for monthly data is what one would know after observing a Bansal-Yaron economy for 417 years.
- Intermediate scale for annual data is what one would know after observing a Bansal-Yaron economy for 1600 years.
- Values shown next slide.

## Table 1. The Prior Distribution of the Parameters of the Law of Motion of the Pricing Kernel

| | Monthly | | Annual | |
|---|---|---|---|---|
| Parameter $\eta$ | Location | Scale | Location | Scale |
| $a_1$ | -0.00776 | 0.02203 | -0.03835 | 0.04292 |
| $a_2$ | 0.05157 | 0.04670 | 0.04199 | 0.03535 |
| $a_3$ | 0.01967 | 0.00722 | 0.00065 | 0.01323 |
| $a_4$ | 0.03365 | 0.00941 | 0.07604 | 0.01354 |
| $b_0$ | 0.01684 | 0.03800 | 0.08739 | 0.07855 |
| $b_1$ | -0.02091 | 0.01470 | 0.00002 | 0.02661 |
| $r_0$ | 0.09952 | 0.01374 | 0.48020 | 0.09367 |
| $r_1$ | 0.24192 | 0.02239 | 0.32158 | 0.05073 |
| $r_2$ | 0.95979 | 0.00432 | 0.77915 | 0.08241 |

Shown are the location and scale of the prior distribution on the parameters $\eta_i$ of the SNP transition density that describes the law of motion of the logarithm of the pricing kernel. The prior for the monthly sampling rate is the prior that one would form after observing a Bansal-Yaron (2004) economy at the parameter settings of Kiku (2006) for 417 years. The prior for the annual sampling rate is the prior that one would form after 1600 years of observation. Before applying these priors, data are normalized by $y_t = (\log M_{t,t+t} + 0.0149120)/0.0251032$ for the monthly priors and $y_t = (\log M_{t,t+t} + 0.162398)/0.290158$ for the annual. The priors used in the analysis are a normal prior with location as shown and scale divided by ten, a normal prior with location and scale as shown, and a normal prior with scale multiplied by ten; called the tight, intermediate, and loose priors, respectively.

# Likelihood Derivation – 1 of 4

- Consider a general set of moment conditions

$$\bar{m}_n = \frac{1}{n}\sum_{t=1}^{n} m_t(y_t, \theta_t) \qquad m_t \in \Re^K$$

  that a scientific theory implies ought to have expectation zero where both $y_t$ and $\theta_t$ are regarded as random

- To estimate the variance when the $m_t(y_t, x_t, \theta^o)$ are uncorrelated, use

$$W_n = \frac{1}{n}\sum_{t=1}^{n} \left[m_t(y_t, \theta_t) - \bar{m}_n\right]\left[m_t(y_t, \theta_t) - \bar{m}_n\right]'$$

- When correlated, use HAC.

- Rely on CLT for normality of $z = W_n^{-1/2}\sqrt{n}\,\bar{m}_n$

# Likelihood Derivation – 2 of 4

- View as a data summary: $z = Z(Y, \theta)$

$$Z : (Y, \theta) \mapsto \sqrt{n} W^{-1/2}{}_n \bar{m}_n$$

where $Y = [y_1|y_2|\cdots|y_n]$, $\theta = [\theta_1|\theta_2|\cdots|\theta_n]$

- $\mathcal{Y}$ is all possible $Y$

- Information loss: Probability can only be assigned to sets $A$ expressed in terms of the original data $(Y, \theta)$ that are preimages $A = Z^{-1}(B)$ of a Borel set $B$ in $\Re^K$.

- Assignment formula:

$$P(A) = P_Z(B) = \int \cdots \int I_B(z)\,(2\pi)^{-K/2} \exp(-z'z/2)\, dz_1 \cdots dz_K$$

# Likelihood Derivation – 3 of 4

- Let $Z_\theta$ denote the map $Y \mapsto Z(Y, \theta)$ for $\theta$ fixed.

- The sets $C$ expressed in terms of $Y$ to which we can assign conditional probability knowing that $\theta$ has occurred have the form $C = Z_\theta^{-1}(B_\theta)$ for some Borel set $B_\theta \in \Re^K$.

- The principle of conditioning is that one assigns conditional probability proportionately to joint probability.

- The constant of proportionality is $1/P(\Re_\theta)$, where $\Re_\theta$ is the Borel set for which $\mathcal{Y} = Z_\theta^{-1}(\Re_\theta)$.

- The conditional probability of $C$ is computed as

$$P(C|\theta) = \frac{P_Z(B_\theta)}{P_Z(\Re_\theta)}$$

# Likelihood Derivation – 4 of 4

- Conditional probability is computed as

$$P(C|\theta) = \frac{1}{P_Z(\Re_\theta)} \int \cdots \int I_{B_\theta}(z)\,(2\pi)^{-K/2} \exp(-z'z/2)\,dz_1 \cdots dz_K$$

- The data summary is

$$z = [W_n(\theta)]^{-1/2} \sqrt{n}\, m_n(\theta)$$

- In our application $P_Z(\Re_\theta) = 1$

- Conclude that the likelihood is

$$\mathcal{L}(\theta) \propto \exp\left\{ -\frac{n}{2} m_n(\theta)'[W_n(\theta)]^{-1} m_n(\theta) \right\}$$

- Same argument as Fisher (1930) used to define fiducial probability. Is Bayesian inference for continuously updated GMM.

# Implementation of $\mathcal{L}(\theta)$ (monthly data)

- $s_t$: Fama-French gross returns $S11 - S54$
  $b_t$: T-debt gross returns t30ret, b1ret, and b10ret.

- Parameters $\theta = \theta_1, \ldots, \theta_{551}$

- Euler equation errors
  $e_t(\theta) = e_t(s_{t+1}, b_{t+1}, \theta_{t+1}) = 1 \; - \; \theta_{t+1} \begin{pmatrix} s_{t+1} \\ b_{t+1} \end{pmatrix}$

- Instruments: $Z_t = (s_t - 1,\, b_t - 1,\, \log(cg_t) - 1,\, \log(lg_t) - 1,\, 1)$

- Moment conditions: $t = 1, \ldots, n = 550$

$$m_t(\theta) = m_t(s_t, b_t, s_{t+1}, b_{t+1}, \theta_{t+1}) = Z_t \otimes e_t(s_{t+1}, b_{t+1}, \theta_{t+1})$$

- Length of $m_t(\theta)$: $K = 756$
  Number of overidentifying restrictions: 206.

# Computing the Weighting Matrix $W$

- Assume factor structure for the random part of $(s_t, b_t)$.

- $\Sigma_e$: one error common for $s_t$, one error common for $b_t$, one idiosyncratic error for each of $(s_t, b_t)$.

- Strengthen zero correlation condition $\mathcal{E}[Z_{i,t}\, e_{j,t}(s_{t+1}, b_{t+1}, \theta^o_{t+1})] = 0$ to

$$\mathsf{Var}[Z_t \otimes e_t(s_{t+1}, b_{t+1}, \theta^o_{t+1})] = \Sigma_z \otimes \Sigma_e,$$

- Implies $\Sigma_z \otimes \Sigma_e$ is diagonalized by $U_z \otimes U_e$ where $U_z$ and $U_e$ are known, constant, orthogonal matrices: 1000 fold efficiency gain.

- Mean correction when estimating diagonal elements to avoid acceptance of an absurd MCMC proposal due to a large variance estimate

$$s_i(\theta) = \frac{1}{n}\sum_{t=1}^{n}\left(v_{t,i}(\theta) - \frac{1}{n}\sum_{t=1}^{n} v_{t,i}(\theta)\right)^2$$

# Implementation of $\mathcal{L}(\theta)$ (monthly data)

- The likelihood for the pricing kernel $\theta$ is, then,

$$\begin{aligned} \mathcal{L}(\theta) &\propto \exp\left\{-\frac{n}{2} m_n(\theta)'(U_z \otimes U_e) S_n^{-1}(\theta)(U_z \otimes U_e)' m_n(\theta)\right\} \\ m_n(\theta) &= \frac{1}{n}\sum_{t=1}^{n} Z_t \otimes e_t(s_{t+1}, b_{t+1}, \theta_{t+1}), \end{aligned}$$

- Full likelihood $\mathcal{L}(\eta, \theta) \propto \mathcal{L}(\theta) \times \prod_{t=1}^{n} f(\theta_{t+1} | \theta_t, \theta_{t-1}, \ldots, \theta_1, \eta)$.

- $\dim(\eta, \theta) = 560$ implies high correlation in MCMC chain.

- To reduce, sample $30,000$ out of 30 million MCMC draws.

## Technical Prior $p(\theta, \eta)$

- Support condition:
  - ▷ Impose mean reversion on law of motion $f(\theta_t | \theta_{t-1}, \ldots, \theta_1, \eta)$

- Constrain the size of the Euler equation errors to be about the same
  - ▷ Annual data: $P(-0.5 < e_{i,n}(\theta) < 0.5) = 0.95$
  - ▷ Monthly data: $P(-0.05 < e_{i,n}(\theta) < 0.05) = 0.95$

- Require an approximation $P_B = \sum_{t=1}^{n} \theta_t$ to the average price of a risk-free, one period bond to be between 1% and 4% per annum,

$$f(P_B) \propto (1 + \cos(\alpha + \beta P_B)) \quad a < P_B < b$$

where $\beta = 2\pi/(b - a)$, $\alpha = \pi - \beta b$.

# Our Approach

- Pricing kernel: $\theta = (\theta_1, \ldots, \theta_{n+1})$
- Hierarchical likelihood: $\mathcal{L}(\theta, \eta) = \mathcal{L}(\theta) \prod_{t=1}^{n} f(\theta_{t+1}|\theta_t, \ldots, \theta_1, \eta)$
  - ▷ $\mathcal{L}(\theta)$ is a function of returns implied by Euler equations
  - ▷ $f(\cdot|\cdot, \eta)$ is the seminonparametric transition density of the pricing kernel.
- Composite prior: $p(\theta, \eta) = p(\eta) p_T(\theta, \eta)$
  - ▷ $p(\eta)$ prior on hyperparameter $\eta$ : tight, intermediate, loose
  - ▷ $p_T(\theta, \eta)$ imposes technical conditions
- Inference: MCMC
  - ▷ Moments of posterior distribution $P(\theta, \eta)$
  - ▷ Posterior probabilities $P(A) = \int \int I_A(\theta, \eta) P(d\theta, d\eta)$ of events $A$ that correspond to hypotheses

# Empirical Results for $\eta$, Tabular

- Posterior changes moving from tight to loose prior
  - ▷ Tight prior binds, intermediate prior informative, loose prior uninformative
  - ▷ Interpret as evidence against Bansal-Yaron economy.

- Conditional variance decreases
  - ▷ Decrease in the variance parameter $r_0$ partially compensated by an increase in kurtosis parameter $a_4$

- Predictability increases
  - ▷ AR parameter $b_1$ increase
  - ▷ ARCH parameter $r_1$ increases and GARCH parameter $r_2$ decreases

Table 4. Posterior Distribution of the Parameters of the Law of Motion of the Monthly Pricing Kernel

| | Tight Prior | | | Intermediate Prior | | | Loose Prior | | |
|---|---|---|---|---|---|---|---|---|---|
| $\eta$ | Mean | Mode | Std.Dev. | Mean | Mode | Std.Dev. | Mean | Mode | Std.Dev. |
| $a_1$ | -0.0090738 | -0.01053 | 0.0021428 | -0.077195 | -0.069801 | 0.01619 | -0.1009 | -0.050606 | 0.049225 |
| $a_2$ | 0.053431 | 0.052584 | 0.0044374 | 0.054482 | 0.018227 | 0.028107 | -0.083432 | -0.024971 | 0.047376 |
| $a_3$ | 0.019685 | 0.020624 | 0.00071004 | 0.023048 | 0.021965 | 0.0069036 | 0.15101 | 0.040962 | 0.039 |
| $a_4$ | 0.033732 | 0.03377 | 0.00093304 | 0.03959 | 0.03318 | 0.0086597 | 0.18599 | 0.23356 | 0.041314 |
| $b_0$ | 0.016074 | 0.013288 | 0.0037388 | -0.027799 | -0.038383 | 0.03545 | -0.46205 | -0.5447 | 0.1281 |
| $b_1$ | -0.020842 | -0.020651 | 0.0014362 | -0.0166 | -0.013206 | 0.014301 | 0.02608 | 0.11015 | 0.062157 |
| $r_0$ | 0.099588 | 0.098673 | 0.0013623 | 0.10183 | 0.094246 | 0.013287 | 0.24502 | 0.048576 | 0.056492 |
| $r_1$ | 0.24352 | 0.24489 | 0.0020933 | 0.26038 | 0.28599 | 0.012799 | 0.30959 | 0.54743 | 0.023572 |
| $r_2$ | 0.95994 | 0.96019 | 0.0004126 | 0.95868 | 0.95272 | 0.0032045 | 0.9323 | 0.83509 | 0.0086239 |

Shown are the mean, mode, and standard deviation of the posterior distribution of the parameters $\eta_i$ of the SNP transition density (4) that describes the law of motion of the pricing kernel $\theta_t$. Before applying these priors, data are normalized by $y_t = (\log \theta_t + 0.0149120)/0.0251032$ to conform to the conventions of Table 1. The tight prior uses the scaling of Table 1 divided by ten, the intermediate prior uses the scaling of Table 1, and the loose prior uses the scaling of Table 1 multiplied by ten.

Table 5. Posterior Distribution of the Parameters of the Law of Motion of the Annual Pricing Kernel

| | Tight Prior | | | Intermediate Prior | | | Loose Prior | | |
|---|---|---|---|---|---|---|---|---|---|
| $\eta$ | Mean | Mode | Std.Dev. | Mean | Mode | Std.Dev. | Mean | Mode | Std.Dev. |
| $a_1$ | -0.039511 | -0.033577 | 0.004254 | -0.099579 | -0.097374 | 0.035642 | -0.42682 | -0.38172 | 0.21314 |
| $a_2$ | 0.042694 | 0.042747 | 0.0035054 | 0.0639 | 0.080284 | 0.033432 | 0.045261 | 0.083427 | 0.21359 |
| $a_3$ | 0.00065929 | 0.001091 | 0.0013239 | 0.0045749 | 0.0051956 | 0.012995 | 0.051852 | 0.40411 | 0.11287 |
| $a_4$ | 0.076104 | 0.076042 | 0.0013435 | 0.076267 | 0.093651 | 0.013435 | 0.07014 | 0.27934 | 0.1279 |
| $b_0$ | 0.085887 | 0.087822 | 0.007847 | 0.015376 | 0.0018082 | 0.074239 | 0.035331 | -0.33724 | 0.42591 |
| $b_1$ | 0.00013013 | 0.0022964 | 0.0026463 | 0.0089322 | 0.0048752 | 0.026314 | 0.12583 | 0.31019 | 0.15749 |
| $r_0$ | 0.48361 | 0.47112 | 0.0093066 | 0.54138 | 0.47095 | 0.084174 | 0.95554 | 0.1963 | 0.25558 |
| $r_1$ | 0.32297 | 0.31582 | 0.0050372 | 0.34567 | 0.3993 | 0.047824 | 0.33342 | 0.8081 | 0.18451 |
| $r_2$ | 0.7839 | 0.78724 | 0.0081781 | 0.82938 | 0.7672 | 0.048072 | 0.43532 | 0.30476 | 0.21749 |

Shown are the mean, mode, and standard deviation of the posterior distribution of the parameters $\eta_i$ of the SNP transition density (4) that describes the law of motion of the pricing kernel $\theta_t$. Before applying these priors, data are normalized by $y_t = (\log \theta_t + 0.162398)/0.290158$ to conform to the conventions of Table 1. The tight prior uses the scaling of Table 1 divided by ten, the intermediate prior uses the scaling of Table 1, and the loose prior uses the scaling of Table 1 multiplied by ten.

# Empirical Results for $\eta$, Graphical

- Plot conditional density $f(\theta_{t+1}|\theta_t, \ldots, \theta_1, \eta)$ for historical conditioning sets $(\theta_t, \ldots, \theta_1, \eta)$

- Large changes moving from tight to loose prior under all conditions

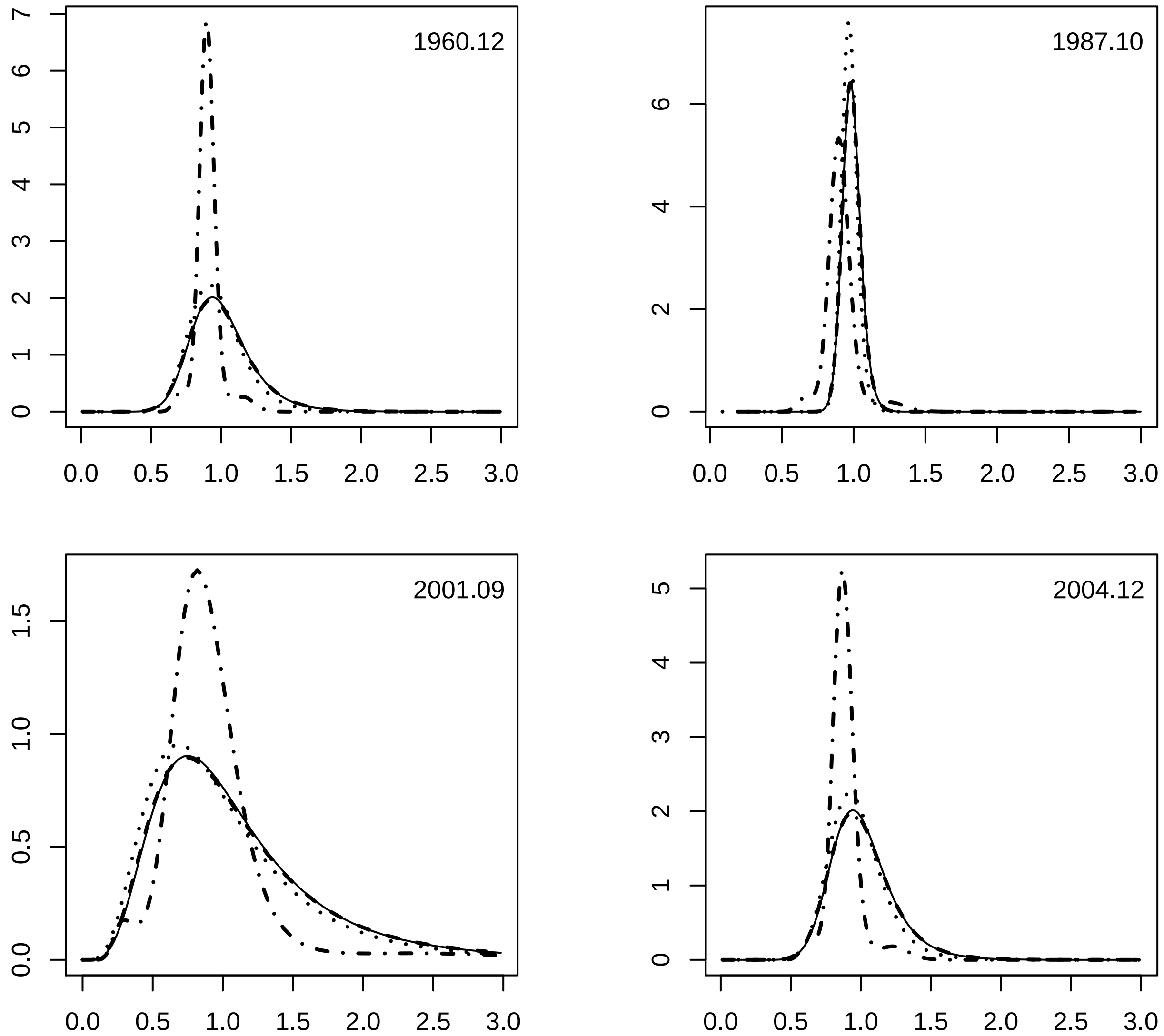
- Features missed by Bansal-Yaron economy
  - ▷ Conditional variance too large
  - ▷ Location and scale do not move enough

- Bansal-Yaron economy misses some predictability of the pricing kernel

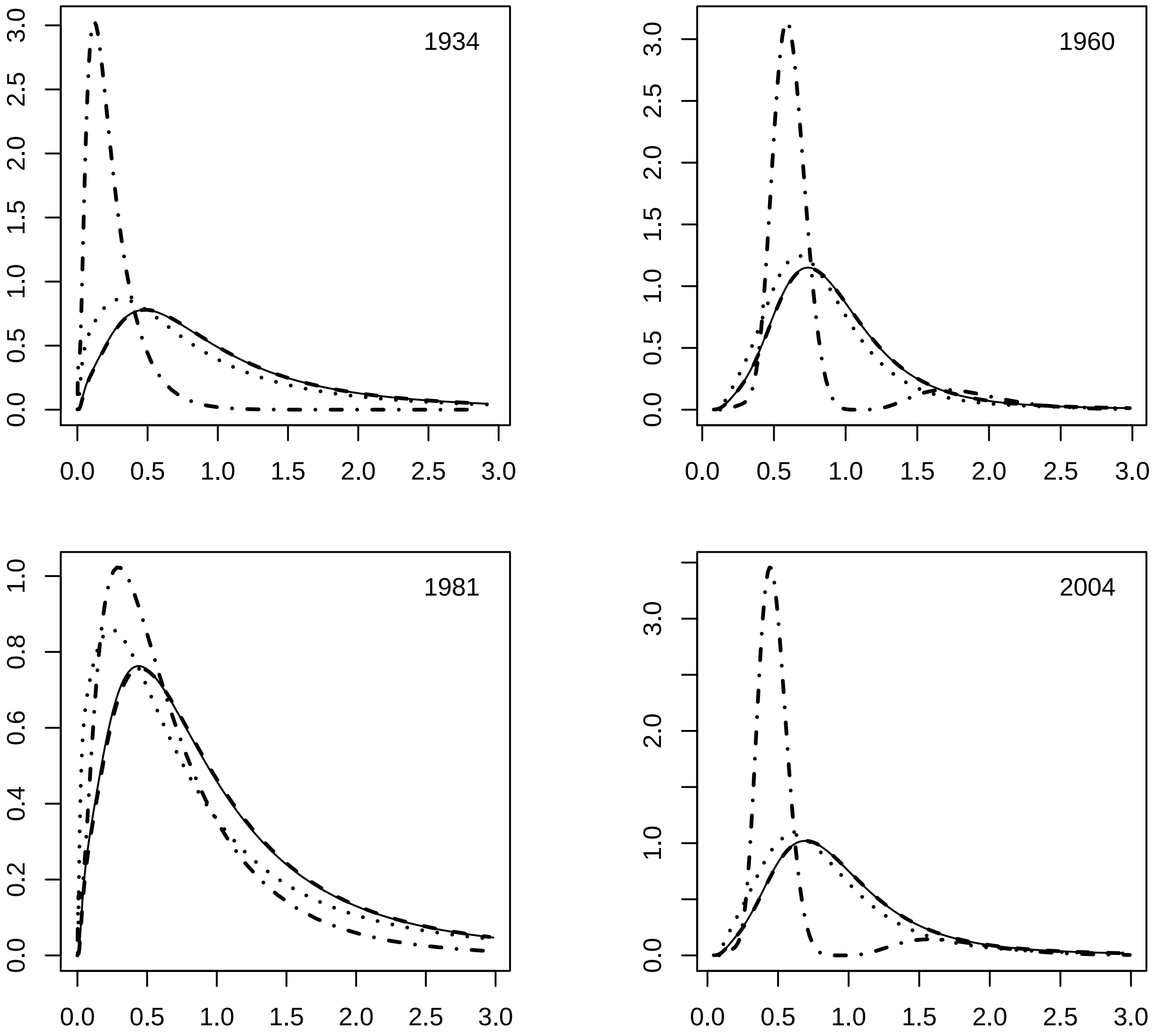
# Legend for Next Two Figures

Plotted is the conditional density $f(\theta_{t+1}|\theta_t, \ldots, \theta_1)$ of the pricing kernel conditional on $\theta_t$ up to the year and month shown in the upper right of each panel. The solid line shows the density with $\eta$ set to the location parameter of the prior in Table 1. The dashed line has $\eta$ set to the mode of the posterior distribution under the tight prior. The dotted line has $\eta$ set to the mode of the posterior distribution under the intermediate prior. The dotted-dashed line has $\eta$ set to the mode of the posterior distribution under the loose prior. The conditioning set is the posterior mean of $\theta_t$ under the tight prior in each case.

# Figure 1. The Law of Motion of the Monthly Pricing Kernel



# Figure 2. The Law of Motion of the Annual Pricing Kernel



# Table 6. Posterior Probabilities of the Models

| Prior | Probability | |
|---|---|---|
| | Monthly Data | Annual Data |
| Tight | 2.2e-9 | 0.0004 |
| Intermediate | 0.0025 | 0.0224 |
| Loose | 0.9975 | 0.9772 |

Shown are the posterior probabilities of the models under the tight, intermediate, and loose priors computed using Newton and Raftery's (1994) $\widehat{p}_4$ method for computing the marginal likelihood from an MCMC chain and assigning equal prior probability to each model.

# Figure 3. Hansen-Jagannathan Bounds for the Monthly Pricing Kernel

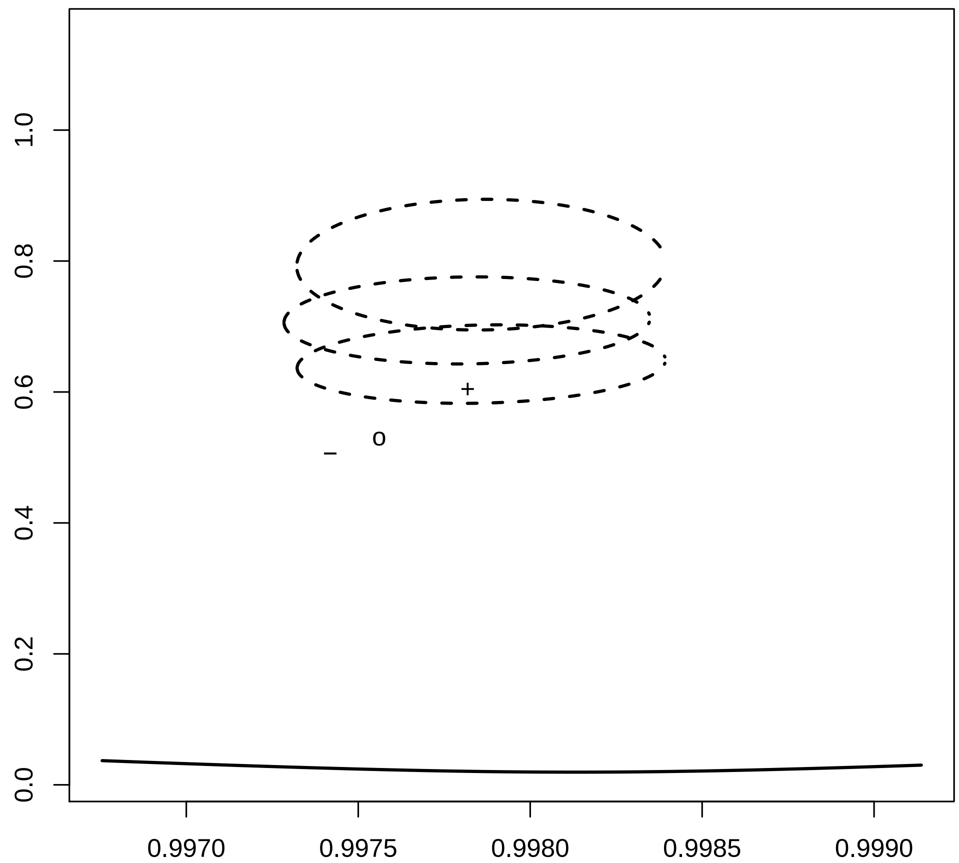
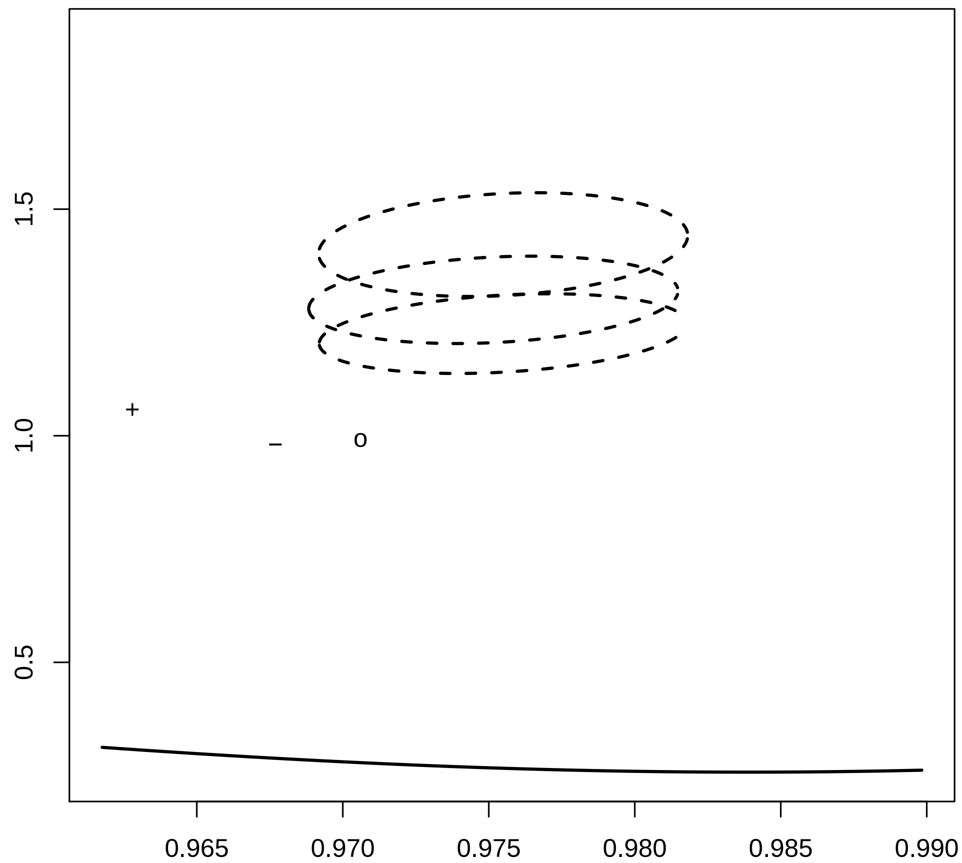


Figure 4. Hansen-Jagannathan Bounds for the Annual Pricing Kernel



# Sample Paths $\{\theta_t\}_{t=1}^n$ of the Pricing Kernel

- Very different signal to noise ratio between annual and monthly data

$$70\% \quad \text{vs} \quad 1\% \quad \text{of} \quad |\bar{\theta}_t/\sqrt{Var(\theta_t)}| > 1$$

- Are periods of extreme volatility: 1970–1975, 2000–2003

- 1981, 1982: $\bar{\theta}_t$ much higher for annual than monthly data
  - ▷ Possibly due to differences in debt portfolios
  - ▷ Pre 1960 events may influence post 1960 annual path

# Figure 5. The Posterior Mean of the Monthly Pricing Kernel

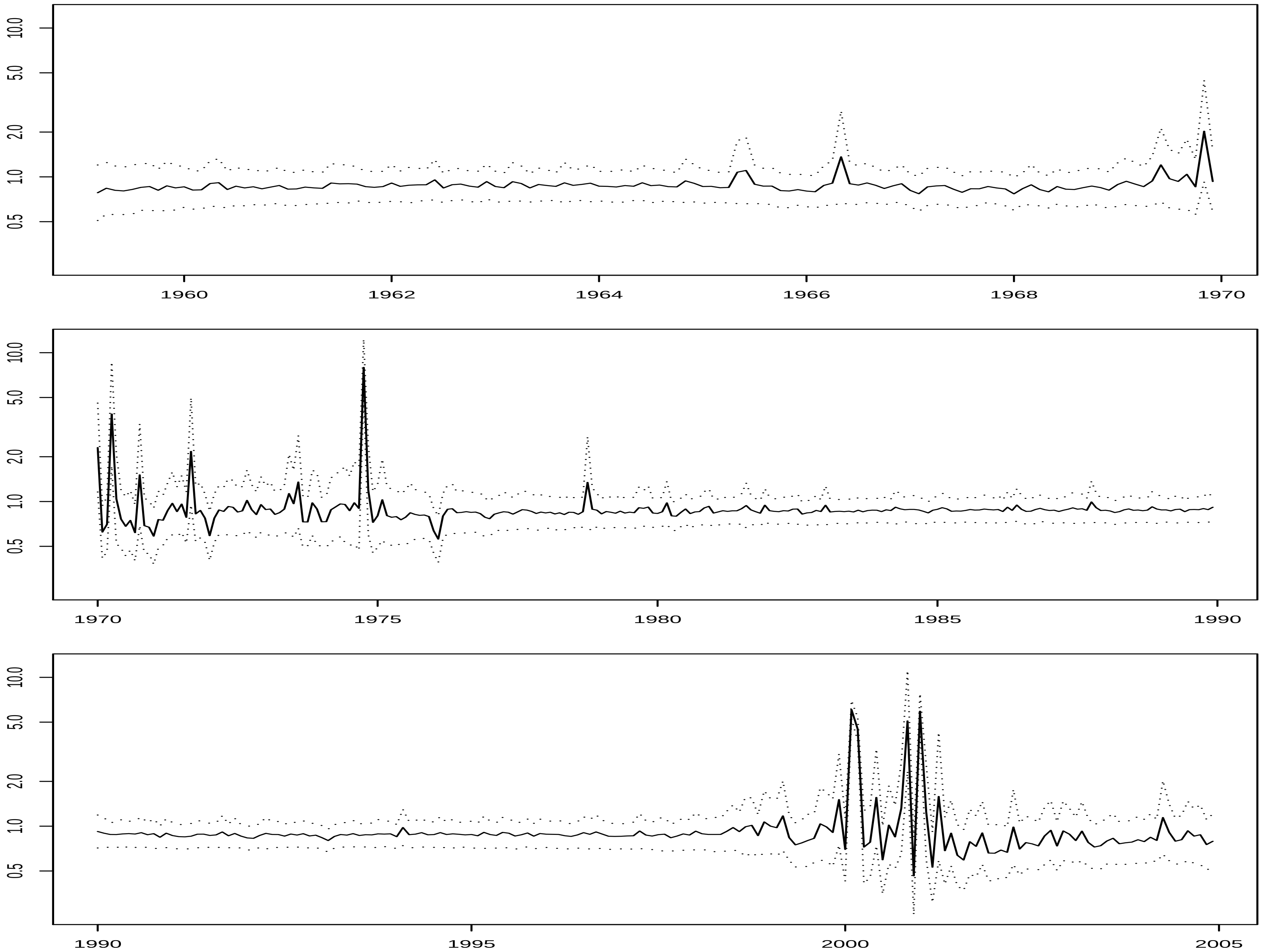
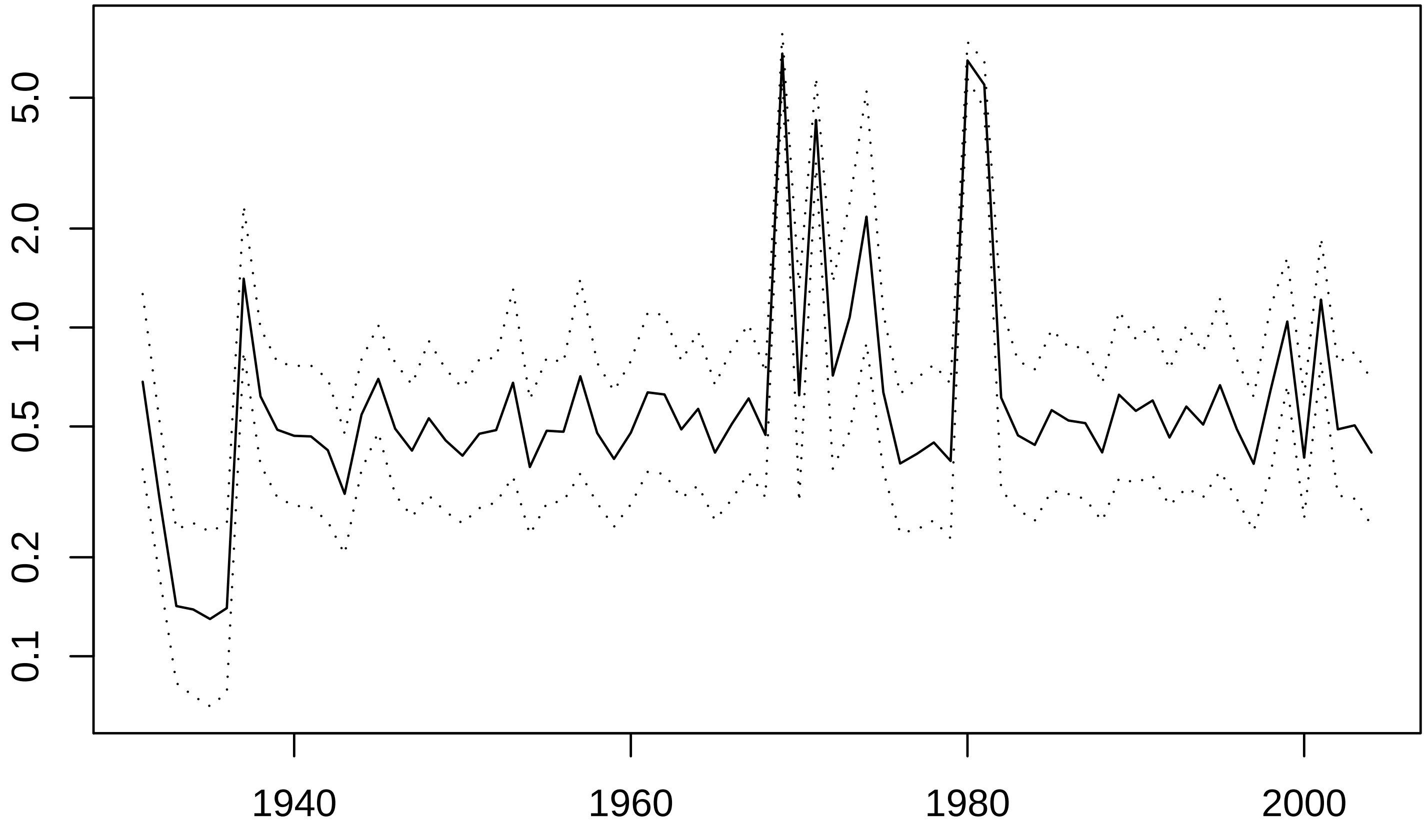


Figure 6. The Posterior Mean of the Annual Pricing Kernel



# Regression Implications of Recursive Utility

- Regression $R^2$
  1. $R^2 = 1$ in log regression $\log(\theta_{t+1}) = c_1 + c_2 \log\left(\frac{C_{t+1}}{C_t}\right) + c_3 \log\left(R_{c,t+1}\right)$
  2. $R^2 < 1$ if $C_t$ and $R_{ct}$ are measured with error
  3. $R^2 < 1$ if $R_{ct}$ omitted or a proxy substituted

- Suggests three checks for recursive plausibility
  1. Posterior $R^2$ of $\log(\theta_{t+1})$ on $\log\left(C_{t+1}/C_t\right)$
  2. Posterior $R^2$ of $\log(\theta_{t+1})$ on $\log\left(C_{t+1}/C_t\right)$ and market return
  3. Posterior $R^2$ of $\log(\theta_{t+1})$ on $\log\left(C_{t+1}/C_t\right)$ and $\log R_{ct}$.

Table 11. Posterior Probabilities of Regression $R^2$ for the Monthly Pricing Kernel

| Prior | Measurement Error | | | |
|---|---|---|---|---|
| | 1% | 2% | 5% | 10% |
| | Log Pricing Kernel on Log Consumption Growth | | | |
| Tight | <0.01 | 0.60 | 0.99 | 0.99 |
| Intermediate | <0.01 | 0.60 | 0.99 | 0.99 |
| Loose | <0.01 | 0.60 | 0.99 | 0.99 |
| | Log Pricing Kernel on Log Consumption Growth and Log Market Return | | | |
| Tight | <0.01 | <0.01 | <0.01 | <0.01 |
| Intermediate | <0.01 | <0.01 | <0.01 | <0.01 |
| Loose | <0.01 | <0.01 | <0.01 | <0.01 |
| | Log Pricing Kernel on Log Consumption Growth and Log Wealth Return | | | |
| Tight | 0.15 | 0.99 | 0.99 | 0.99 |
| Intermediate | 0.20 | 0.99 | 0.99 | 0.99 |
| Loose | 0.55 | 0.99 | 0.99 | 0.99 |

Table 12. Posterior Probabilities of Regression $R^2$ for the Annual Pricing Kernel

| Prior | Measurement Error 1% | 2% | 5% | 10% |
|---|---|---|---|---|
| | Log Pricing Kernel on Log Consumption Growth | | | |
| Tight | <0.01 | 0.20 | 0.90 | 0.99 |
| Intermediate | <0.01 | 0.10 | 0.85 | 0.95 |
| Loose | <0.01 | 0.10 | 0.75 | 0.95 |
| | Log Pricing Kernel on Log Consumption Growth and Log Market Return | | | |
| Tight | <0.01 | <0.01 | <0.01 | <0.01 |
| Intermediate | <0.01 | <0.01 | <0.01 | <0.01 |
| Loose | <0.01 | <0.01 | <0.01 | <0.01 |
| | Log Pricing Kernel on Log Consumption Growth and Log Wealth Return | | | |
| Tight | <0.01 | <0.01 | 0.80 | 0.99 |
| Intermediate | <0.01 | <0.01 | 0.40 | 0.99 |
| Loose | <0.01 | <0.01 | 0.50 | 0.90 |
| | Log Pricing Kernel on Log Consumption Growth and Log Wealth Return Years 1960–2004 only | | | |
| Tight | <0.01 | 0.60 | 0.99 | 0.99 |
| Intermediate | <0.01 | 0.25 | 0.99 | 0.99 |
| Loose | <0.01 | 0.20 | 0.99 | 0.99 |

# Conclusions

- Bansal-Yaron consumption dynamics mildly misspecified
  - ▷ First check a joint test of consumption dynamics and recursive
  - ▷ First check accepts

- Bansal-Yaron cash flow dynamics more seriously misspecified
  - ▷ Second check a joint test of cash flow dynamics and recursive
  - ▷ Second check rejects

- Recursive utility plausible
  - ▷ Third check uses $R_{ct}$ computed independently of Bansal-Yaron dynamics
  - ▷ Third check accepts

Table 7. Regression $R^2$ for the Monthly Pricing Kernel in a Bansal-Yaron Economy

| | Measurement Error | | | | |
|---|---|---|---|---|---|
| Lag | 0% | 1% | 2% | 5% | 10% |
| | **Log Pricing Kernel on Log Consumption Growth** | | | | |
| 0 | 0.1616 | 0.0509 | 0.0169 | 0.0032 | 0.0007 |
| 1 | 0.1622 | 0.0510 | 0.0170 | 0.0036 | 0.0010 |
| 2 | 0.1623 | 0.0512 | 0.0171 | 0.0036 | 0.0011 |
| 3 | 0.1626 | 0.0512 | 0.0171 | 0.0036 | 0.0011 |
| 4 | 0.1627 | 0.0513 | 0.0171 | 0.0038 | 0.0011 |
| 5 | 0.1629 | 0.0513 | 0.0172 | 0.0038 | 0.0012 |
| 6 | 0.1629 | 0.0514 | 0.0172 | 0.0038 | 0.0012 |
| 7 | 0.1629 | 0.0514 | 0.0172 | 0.0038 | 0.0012 |
| 8 | 0.1631 | 0.0514 | 0.0172 | 0.0038 | 0.0012 |
| 9 | 0.1631 | 0.0515 | 0.0173 | 0.0039 | 0.0012 |
| 10 | 0.1637 | 0.0516 | 0.0173 | 0.0039 | 0.0014 |
| 11 | 0.1639 | 0.0514 | 0.0178 | 0.0039 | 0.0014 |
| | **Log Pricing Kernel on Log Consumption Growth and Log Market Return** | | | | |
| 0 | 0.5946 | 0.5346 | 0.4417 | 0.2207 | 0.0772 |
| 1 | 0.5950 | 0.5346 | 0.4418 | 0.2207 | 0.0774 |
| 2 | 0.5952 | 0.5347 | 0.4418 | 0.2208 | 0.0775 |
| 3 | 0.5953 | 0.5348 | 0.4419 | 0.2208 | 0.0776 |
| 4 | 0.5954 | 0.5348 | 0.4419 | 0.2209 | 0.0776 |
| 5 | 0.5958 | 0.5349 | 0.4420 | 0.2210 | 0.0778 |
| 6 | 0.5958 | 0.5349 | 0.4420 | 0.2210 | 0.0780 |
| 7 | 0.5958 | 0.5349 | 0.4420 | 0.2212 | 0.0782 |
| 8 | 0.5961 | 0.5351 | 0.4420 | 0.2214 | 0.0783 |
| 9 | 0.5962 | 0.5352 | 0.4420 | 0.2214 | 0.0784 |
| 10 | 0.5964 | 0.5352 | 0.4421 | 0.2214 | 0.0784 |
| 11 | 0.5965 | 0.5354 | 0.4424 | 0.2215 | 0.0788 |
| | **Log Pricing Kernel on Log Consumption Growth and Log Wealth Return** | | | | |
| 0 | 1.0000 | 0.3217 | 0.1204 | 0.0193 | 0.0065 |
| 1 | 1.0000 | 0.3218 | 0.1206 | 0.0197 | 0.0070 |
| 2 | 1.0000 | 0.3222 | 0.1207 | 0.0198 | 0.0072 |
| 3 | 1.0000 | 0.3222 | 0.1207 | 0.0199 | 0.0072 |
| 4 | 1.0000 | 0.3222 | 0.1207 | 0.0201 | 0.0072 |
| 5 | 1.0000 | 0.3223 | 0.1208 | 0.0201 | 0.0073 |
| 6 | 1.0000 | 0.3225 | 0.1208 | 0.0201 | 0.0073 |
| 7 | 1.0000 | 0.3225 | 0.1208 | 0.0201 | 0.0074 |
| 8 | 1.0000 | 0.3227 | 0.1208 | 0.0202 | 0.0074 |
| 9 | 1.0000 | 0.3228 | 0.1209 | 0.0203 | 0.0075 |
| 10 | 1.0000 | 0.3230 | 0.1210 | 0.0204 | 0.0077 |
| 11 | 1.0000 | 0.3230 | 0.1214 | 0.0204 | 0.0077 |

Table 8. Regression $R^2$ for the Annual Pricing Kernel in a Bansal-Yaron Economy

| | Measurement Error | | | | |
|---|---|---|---|---|---|
| Lag | 0% | 1% | 2% | 5% | 10% |
| | **Log Pricing Kernel on Log Consumption Growth** | | | | |
| 0 | 0.2214 | 0.2041 | 0.1331 | 0.0406 | 0.0095 |
| 1 | 0.2308 | 0.2092 | 0.1368 | 0.0409 | 0.0103 |
| 2 | 0.2453 | 0.2168 | 0.1414 | 0.0410 | 0.0109 |
| 3 | 0.2564 | 0.2267 | 0.1494 | 0.0410 | 0.0110 |
| 4 | 0.2585 | 0.2306 | 0.1503 | 0.0419 | 0.0110 |
| | **Log Pricing Kernel on Log Consumption Growth and Log Market Return** | | | | |
| 0 | 0.5931 | 0.5866 | 0.5674 | 0.4922 | 0.3921 |
| 1 | 0.6036 | 0.5950 | 0.5750 | 0.4949 | 0.3946 |
| 2 | 0.6081 | 0.5974 | 0.5767 | 0.4953 | 0.3951 |
| 3 | 0.6176 | 0.6048 | 0.5824 | 0.4954 | 0.3956 |
| 4 | 0.6194 | 0.6082 | 0.5832 | 0.4965 | 0.3966 |
| | **Log Pricing Kernel on Log Consumption Growth and Log Wealth Return** | | | | |
| 0 | 1.0000 | 0.7768 | 0.5386 | 0.2529 | 0.0631 |
| 1 | 1.0000 | 0.7794 | 0.5428 | 0.2547 | 0.0641 |
| 2 | 1.0000 | 0.7822 | 0.5488 | 0.2562 | 0.0648 |
| 3 | 1.0000 | 0.7849 | 0.5562 | 0.2565 | 0.0648 |
| 4 | 1.0000 | 0.7871 | 0.5581 | 0.2573 | 0.0651 |

Table 9. Posterior Distribution of Regression $R^2$ for the Monthly Pricing Kernel

| | Tight Prior | | | | | Intermediate Prior | | | | | Loose Prior | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Lag | 5% | 25% | 50% | 75% | 95% | 5% | 25% | 50% | 75% | 95% | 5% | 25% | 50% | 75% | 95% |
| Regression of Log Pricing Kernel on Log Consumption Growth | | | | | | | | | | | | | | | |
| 0 | 5.9e-6 | 0.0001 | 0.0006 | 0.0017 | 0.0051 | 6.6e-6 | 0.0002 | 0.0007 | 0.0019 | 0.0052 | 5.7e-6 | 0.0001 | 0.0006 | 0.0016 | 0.0045 |
| 1 | 0.0001 | 0.0008 | 0.0019 | 0.0038 | 0.0081 | 0.0002 | 0.0008 | 0.0020 | 0.0039 | 0.0085 | 0.0001 | 0.0006 | 0.0015 | 0.0031 | 0.0065 |
| 2 | 0.0005 | 0.0020 | 0.0040 | 0.0070 | 0.0140 | 0.0007 | 0.0024 | 0.0045 | 0.0077 | 0.0143 | 0.0004 | 0.0016 | 0.0031 | 0.0054 | 0.0103 |
| 3 | 0.0013 | 0.0034 | 0.0059 | 0.0095 | 0.0162 | 0.0015 | 0.0039 | 0.0067 | 0.0107 | 0.0186 | 0.0011 | 0.0031 | 0.0053 | 0.0085 | 0.0144 |
| 4 | 0.0020 | 0.0047 | 0.0075 | 0.0114 | 0.0183 | 0.0022 | 0.0050 | 0.0081 | 0.0123 | 0.0204 | 0.0018 | 0.0042 | 0.0067 | 0.0101 | 0.0164 |
| 5 | 0.0028 | 0.0059 | 0.0091 | 0.0131 | 0.0209 | 0.0030 | 0.0062 | 0.0097 | 0.0141 | 0.0230 | 0.0023 | 0.0051 | 0.0079 | 0.0114 | 0.0178 |
| 6 | 0.0040 | 0.0078 | 0.0113 | 0.0157 | 0.0241 | 0.0041 | 0.0078 | 0.0116 | 0.0166 | 0.0257 | 0.0035 | 0.0068 | 0.0100 | 0.0139 | 0.0213 |
| 7 | 0.0048 | 0.0091 | 0.0127 | 0.0173 | 0.0261 | 0.0051 | 0.0092 | 0.0131 | 0.0183 | 0.0276 | 0.0044 | 0.0080 | 0.0115 | 0.0156 | 0.0232 |
| 8 | 0.0058 | 0.0103 | 0.0141 | 0.0188 | 0.0279 | 0.0061 | 0.0104 | 0.0147 | 0.0198 | 0.0294 | 0.0053 | 0.0091 | 0.0128 | 0.0172 | 0.0250 |
| 9 | 0.0069 | 0.0116 | 0.0158 | 0.0210 | 0.0310 | 0.0071 | 0.0119 | 0.0162 | 0.0216 | 0.0311 | 0.0061 | 0.0104 | 0.0142 | 0.0188 | 0.0270 |
| 10 | 0.0079 | 0.0129 | 0.0172 | 0.0225 | 0.0322 | 0.0082 | 0.0132 | 0.0177 | 0.0233 | 0.0329 | 0.0071 | 0.0117 | 0.0157 | 0.0204 | 0.0283 |
| 11 | 0.0095 | 0.0149 | 0.0197 | 0.0253 | 0.0361 | 0.0096 | 0.0153 | 0.0201 | 0.0258 | 0.0366 | 0.0093 | 0.0150 | 0.0199 | 0.0254 | 0.0353 |
| Regression of Log Pricing Kernel on Log Consumption Growth and Log Market Return | | | | | | | | | | | | | | | |
| 0 | 0.0002 | 0.0013 | 0.0030 | 0.0057 | 0.0112 | 0.0005 | 0.0021 | 0.0043 | 0.0076 | 0.0138 | 0.0005 | 0.0023 | 0.0046 | 0.0079 | 0.0142 |
| 1 | 0.0016 | 0.0039 | 0.0065 | 0.0101 | 0.0167 | 0.0020 | 0.0047 | 0.0077 | 0.0116 | 0.0194 | 0.0023 | 0.0056 | 0.0091 | 0.0135 | 0.0214 |
| 2 | 0.0035 | 0.0073 | 0.0112 | 0.0158 | 0.0240 | 0.0042 | 0.0083 | 0.0122 | 0.0175 | 0.0270 | 0.0051 | 0.0102 | 0.0149 | 0.0205 | 0.0295 |
| 3 | 0.0055 | 0.0100 | 0.0144 | 0.0195 | 0.0288 | 0.0064 | 0.0113 | 0.0160 | 0.0218 | 0.0319 | 0.0074 | 0.0135 | 0.0186 | 0.0243 | 0.0343 |
| 4 | 0.0076 | 0.0126 | 0.0174 | 0.0229 | 0.0323 | 0.0085 | 0.0139 | 0.0191 | 0.0253 | 0.0358 | 0.0097 | 0.0167 | 0.0225 | 0.0292 | 0.0406 |
| 5 | 0.0099 | 0.0156 | 0.0208 | 0.0268 | 0.0374 | 0.0107 | 0.0169 | 0.0227 | 0.0292 | 0.0400 | 0.0119 | 0.0190 | 0.0251 | 0.0318 | 0.0436 |
| 6 | 0.0122 | 0.0187 | 0.0243 | 0.0306 | 0.0417 | 0.0135 | 0.0201 | 0.0262 | 0.0331 | 0.0441 | 0.0146 | 0.0221 | 0.0286 | 0.0359 | 0.0478 |
| 7 | 0.0146 | 0.0213 | 0.0271 | 0.0339 | 0.0450 | 0.0159 | 0.0231 | 0.0294 | 0.0366 | 0.0482 | 0.0174 | 0.0255 | 0.0323 | 0.0404 | 0.0529 |
| 8 | 0.0176 | 0.0255 | 0.0322 | 0.0393 | 0.0515 | 0.0197 | 0.0279 | 0.0345 | 0.0425 | 0.0548 | 0.0220 | 0.0305 | 0.0374 | 0.0456 | 0.0588 |
| 9 | 0.0199 | 0.0285 | 0.0354 | 0.0429 | 0.0558 | 0.0222 | 0.0308 | 0.0376 | 0.0457 | 0.0586 | 0.0245 | 0.0335 | 0.0410 | 0.0494 | 0.0632 |
| 10 | 0.0246 | 0.0335 | 0.0411 | 0.0494 | 0.0633 | 0.0262 | 0.0355 | 0.0429 | 0.0514 | 0.0651 | 0.0278 | 0.0366 | 0.0443 | 0.0530 | 0.0673 |
| 11 | 0.0277 | 0.0372 | 0.0455 | 0.0540 | 0.0684 | 0.0291 | 0.0392 | 0.0469 | 0.0560 | 0.0699 | 0.0321 | 0.0421 | 0.0504 | 0.0597 | 0.0748 |
| Regression of Log Pricing Kernel on Log Consumption Growth and Log Wealth Return | | | | | | | | | | | | | | | |
| 0 | 0.0679 | 0.0894 | 0.1078 | 0.1262 | 0.1568 | 0.0724 | 0.1070 | 0.1329 | 0.1600 | 0.1990 | 0.0915 | 0.1264 | 0.1554 | 0.1852 | 0.2339 |
| 1 | 0.1139 | 0.1436 | 0.1662 | 0.1917 | 0.2273 | 0.1058 | 0.1440 | 0.1774 | 0.2112 | 0.2534 | 0.1288 | 0.1714 | 0.2061 | 0.2403 | 0.2869 |
| 2 | 0.1387 | 0.1698 | 0.1923 | 0.2179 | 0.2526 | 0.1237 | 0.1638 | 0.1967 | 0.2280 | 0.2725 | 0.1600 | 0.2016 | 0.2336 | 0.2674 | 0.3162 |
| 3 | 0.1552 | 0.1873 | 0.2107 | 0.2356 | 0.2723 | 0.1390 | 0.1810 | 0.2136 | 0.2442 | 0.2885 | 0.1817 | 0.2239 | 0.2559 | 0.2908 | 0.3407 |
| 4 | 0.1692 | 0.2018 | 0.2245 | 0.2498 | 0.2854 | 0.1519 | 0.1929 | 0.2245 | 0.2551 | 0.2996 | 0.1970 | 0.2387 | 0.2701 | 0.3039 | 0.3521 |
| 5 | 0.1797 | 0.2131 | 0.2361 | 0.2605 | 0.2965 | 0.1654 | 0.2058 | 0.2382 | 0.2662 | 0.3120 | 0.2075 | 0.2510 | 0.2840 | 0.3177 | 0.3680 |
| 6 | 0.1896 | 0.2229 | 0.2458 | 0.2706 | 0.3061 | 0.1728 | 0.2125 | 0.2455 | 0.2749 | 0.3205 | 0.2149 | 0.2617 | 0.2944 | 0.3273 | 0.3773 |
| 7 | 0.1991 | 0.2322 | 0.2552 | 0.2791 | 0.3140 | 0.1790 | 0.2213 | 0.2535 | 0.2845 | 0.3286 | 0.2271 | 0.2721 | 0.3036 | 0.3356 | 0.3857 |
| 8 | 0.2084 | 0.2407 | 0.2637 | 0.2885 | 0.3241 | 0.1877 | 0.2299 | 0.2619 | 0.2925 | 0.3388 | 0.2354 | 0.2810 | 0.3126 | 0.3452 | 0.3944 |
| 9 | 0.2138 | 0.2469 | 0.2704 | 0.2954 | 0.3314 | 0.1925 | 0.2356 | 0.2681 | 0.2986 | 0.3457 | 0.2415 | 0.2875 | 0.3208 | 0.3543 | 0.4040 |
| 10 | 0.2184 | 0.2515 | 0.2760 | 0.3008 | 0.3374 | 0.1978 | 0.2409 | 0.2727 | 0.3038 | 0.3507 | 0.2450 | 0.2924 | 0.3256 | 0.3598 | 0.4092 |
| 11 | 0.2241 | 0.2579 | 0.2816 | 0.3071 | 0.3435 | 0.2040 | 0.2475 | 0.2788 | 0.3088 | 0.3558 | 0.2512 | 0.2990 | 0.3321 | 0.3664 | 0.4163 |

Table 10. Posterior Distribution of Regression $R^2$ for the Annual Pricing Kernel

| | Tight Prior | | | | | Intermediate Prior | | | | | Loose Prior | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Lag | 5% | 25% | 50% | 75% | 95% | 5% | 25% | 50% | 75% | 95% | 5% | 25% | 50% | 75% | 95% |
| Regression of Log Pricing Kernel on Log Consumption Growth | | | | | | | | | | | | | | | |
| 0 | 0.0342 | 0.0651 | 0.0915 | 0.1211 | 0.1701 | 0.0211 | 0.0480 | 0.0731 | 0.1021 | 0.1510 | 0.0168 | 0.0405 | 0.0650 | 0.0928 | 0.1425 |
| 1 | 0.0412 | 0.0727 | 0.1002 | 0.1295 | 0.1798 | 0.0287 | 0.0571 | 0.0829 | 0.1118 | 0.1624 | 0.0276 | 0.0518 | 0.0757 | 0.1028 | 0.1500 |
| 2 | 0.0500 | 0.0848 | 0.1139 | 0.1445 | 0.1940 | 0.0381 | 0.0697 | 0.0976 | 0.1285 | 0.1800 | 0.0373 | 0.0660 | 0.0916 | 0.1218 | 0.1699 |
| 3 | 0.0961 | 0.1424 | 0.1784 | 0.2184 | 0.2815 | 0.0750 | 0.1177 | 0.1537 | 0.1923 | 0.2552 | 0.0709 | 0.1102 | 0.1443 | 0.1837 | 0.2472 |
| 4 | 0.1012 | 0.1479 | 0.1864 | 0.2272 | 0.2920 | 0.0810 | 0.1248 | 0.1611 | 0.2011 | 0.2642 | 0.0768 | 0.1184 | 0.1532 | 0.1956 | 0.2603 |
| Regression of Log Pricing Kernel on Log Consumption Growth and Log Market Return | | | | | | | | | | | | | | | |
| 0 | 0.0920 | 0.1373 | 0.1690 | 0.2057 | 0.2601 | 0.0724 | 0.1140 | 0.1476 | 0.1838 | 0.2411 | 0.0710 | 0.1111 | 0.1426 | 0.1783 | 0.2323 |
| 1 | 0.1158 | 0.1613 | 0.1959 | 0.2345 | 0.2905 | 0.0904 | 0.1348 | 0.1693 | 0.2063 | 0.2664 | 0.0848 | 0.1243 | 0.1569 | 0.1951 | 0.2526 |
| 2 | 0.1368 | 0.1839 | 0.2215 | 0.2594 | 0.3181 | 0.1088 | 0.1566 | 0.1940 | 0.2326 | 0.2944 | 0.0999 | 0.1432 | 0.1775 | 0.2166 | 0.2764 |
| 3 | 0.1808 | 0.2355 | 0.2771 | 0.3189 | 0.3843 | 0.1471 | 0.2002 | 0.2406 | 0.2847 | 0.3532 | 0.1288 | 0.1800 | 0.2182 | 0.2617 | 0.3331 |
| 4 | 0.1911 | 0.2460 | 0.2876 | 0.3317 | 0.3980 | 0.1586 | 0.2129 | 0.2541 | 0.2983 | 0.3680 | 0.1402 | 0.1915 | 0.2313 | 0.2740 | 0.3461 |
| Regression of Log Pricing Kernel on Log Consumption Growth and Log Wealth Return | | | | | | | | | | | | | | | |
| 0 | 0.2133 | 0.2678 | 0.3104 | 0.3549 | 0.4190 | 0.1482 | 0.2015 | 0.2419 | 0.2823 | 0.3430 | 0.1436 | 0.2062 | 0.2524 | 0.3031 | 0.3837 |
| 1 | 0.2851 | 0.3482 | 0.3937 | 0.4402 | 0.5092 | 0.2339 | 0.2961 | 0.3396 | 0.3857 | 0.4570 | 0.2144 | 0.2825 | 0.3329 | 0.3877 | 0.4698 |
| 2 | 0.3466 | 0.4098 | 0.4568 | 0.5022 | 0.5692 | 0.3033 | 0.3659 | 0.4119 | 0.4574 | 0.5237 | 0.2752 | 0.3459 | 0.3962 | 0.4492 | 0.5262 |
| 3 | 0.3758 | 0.4398 | 0.4860 | 0.5308 | 0.5960 | 0.3394 | 0.4049 | 0.4520 | 0.4976 | 0.5618 | 0.3222 | 0.3933 | 0.4414 | 0.4929 | 0.5660 |
| 4 | 0.4058 | 0.4754 | 0.5217 | 0.5700 | 0.6346 | 0.3721 | 0.4403 | 0.4884 | 0.5349 | 0.6018 | 0.3576 | 0.4319 | 0.4838 | 0.5365 | 0.6053 |

Shown are quantiles of the posterior distribution of $R^2$ for regressions of the log pricing kernel on lags of log consumption growth, log consumption growth and log stock returns, and log consumption growth and log wealth return under the tight, intermediate, and loose priors.