These pages are a direct copy of a Zoom lecture on 10/28/20. They first show how to solve the permit trading problem for the case where the cost function is $C(Q)=Q$ and output is bounded by 20. They next show how to solve the permit trading problem when $C(Q)=Q^2$ and output is not bounded.

$P = 40$

$C(Q) = Q$

$MAC_1(A_1) = 6A_1$

$MAC_2(A_2) = 2A_2$

$A = Q_1 + Q_2 - 20$

$P_x = 6A_1 = 2A_2 \Rightarrow A_2 = 3A_1$

$A = A_1 + A_2 = A_1 + 3A_1 = 4A_1$

$A_1 = \frac{1}{4}A \quad A_2 = \frac{3}{4}A$

$P_x = \frac{6}{4}A = \frac{3}{2}A = \frac{3}{2}(Q_1 + Q_2 - 20)$

Short cut method

Guess Q_1 Q_2

Compute permit price

Check if total marginal cost

$1 + P_x > = < 20$

We know that $Q_1 = Q_2$

because the marginal condition 1 + Px is the same for both firms.

$Q_1 = 19 \quad Q_2 = 19$

$P_x = \frac{3}{2}(Q_1 + Q_2 - 20) = \frac{3}{2} 19 = 28.5$

$P_x + 1 = 28.5 + 1 = 29.5 < \cancel{4}\,40$

so firms want to increase

$Q_1 = Q_2 = 20$

$A = (Q_1 + Q_2 - 20) = 20$

$P_x = \frac{3}{2} \times 20 = 30$ ✓

$1 + P_x = 1 + 30 = 31 < 40$

firms want to increase but cannot due to 20 limit on output

$A_1 = \frac{1}{4} A = \frac{1}{4} 20 = 5$

Firm 1: $Q_1 = 20$ $A_1 = 5$

total permits $10 + 5 = 15$ ~~20~~

Firm 1 buys $X = 5$ permits from 2

Change problem, idea that
$C(Q) = Q$ and $Q \leq 20$
comes from Kolstead

More realistic

$C(Q) = Q^2$

$MC(Q) = 2Q$

$Q_1 = Q_2 = 14$

$A = Q_1 + Q_2 - 20 = 28 - 20 = 8$

$P_x = \frac{3}{2} A = \frac{3}{2} 8 = 12$

Marginal condition

marginal cost + permit price

$2Q \quad + \quad 12$

$2 \times 14 \quad + 12$

$28 \quad + 12 \quad = 40$

So $Q_1 = Q_2 = 14$ is the answer

$$A_1 = \frac{1}{4} A = \frac{1}{4} 8 = 2$$

$$Q_1 = 14$$

$$Q_1 - \overset{\text{free}}{10} - \overset{A_1}{2} = 14 - 12 = 2$$

Firm 1 short 2 permits, i.e $X = 2$, which it buys from firm 2.

What if permit market collapsed? We'd get competition

$$P = MC(Q)$$

$$40 = 2Q_1$$

$Q_1 = 20$ and

$Q_2 = 20$