Production costs

$$C_1(Q_1) = Q_1^2 + 8$$

$$C_2(Q_1, Q_2) = Q_2^2 + Q_1 Q_2 + 4$$

Marginal costs $\Delta C_i / \Delta Q_i$

$$MC_1(Q_1) = 2Q_1$$

$$MC_2(Q_1, Q_2) = 2Q_2 + Q_1$$

Marginal damage $\Delta C_i / \Delta Q_j$

$$MD_2(Q_1 Q_2) = Q_2$$

What should be produced (merge firms)

$$\Pi(Q_1, Q_2) = P_1 Q_1 + P_2 Q_2 - C_1(Q_1) - C_2(Q_2, Q_2)$$

Set marginal revenue for each product equal to marginal cost

$$P_1 = \Delta C_1 / \Delta Q_1 + \Delta C_2 / \Delta Q_1 = MC_1(Q_1) + MD_2(Q_1, Q_2)$$

$$= 2Q_1 + Q_2$$

$$P_2 = \Delta C_1 / \Delta Q + \Delta C_2 / \Delta Q_2 = 0 + MC_2(Q_1 Q_2)$$

$$= 2Q_2 + Q_1$$

$$P_1 = 2Q_1 + Q_2$$

$$P_2 = 2Q_2 + Q_1$$

or

$$Q_1 = \frac{1}{3}(2P_1 - P_2)$$

$$Q_2 = \frac{1}{3}(2P_2 - P_1)$$

How much should be produced if $P_1 = 11$, $P_2 = 10$

$$Q_1 = \frac{1}{3}(2P_1 - P_2) = \frac{1}{3}(22 - 10) = 4$$

$$Q_2 = \frac{1}{3}(2P_2 - P_1) = \frac{1}{3}(20 - 11) = 3$$

Check to see if solution is viable

$$\Pi_1 = P_1 Q_1 - C_1(Q_1) = 11 \cdot 4 - 4^2 - 8 = 20$$

$$\Pi_2 = P_2 Q_2 - C_2(Q_1 Q_2) = 10 \cdot 3 - 3^2 - 3 \cdot 4 - 4 = 5$$

What will be produced: Each firm sets marginal revenue equal to marginal cost

$$P_1 = MC_1(Q_1)$$

$$P_2 = MC_2(Q_1, Q_2)$$

$$P_1 = 2Q_1$$

$$P_2 = 2Q_2 + Q_1$$

or

$$Q_1 = \frac{1}{2}P_1$$

$$Q_2 = \frac{1}{4}(2P_2 - P_1)$$

How much will be produced if $P_1 = 11$ & $P_2 = 10$

$$Q_1 = 11/2 = 5.5$$

$$Q_2 = \frac{1}{4}(20 - 11) = 9/4 = 2.25$$

} Too much of good 1, to little of good 2

Check to see if viable

$$\Pi_1 = P_1 Q_1 - C_1(Q_1) = 11 \cdot 5.5 - (5.5)^2 - 8 = 22.25$$

$$\Pi_2 = P_2 Q_2 - C_2(Q_1 Q_2) = 10 \cdot 2.25 - (2.25)^2 - (2.25)(5.5) - 4$$

$$= 1.0625$$