

How much will be produced if Firm 1 must pay Firm 2⁶ for damage

The benchmark is Firm 2's profit when there is no damage; i.e., when $Q_1 = 0$

Equating marginal revenue to marginal cost with $Q_1 = 0$

$$P_2 = 2Q_2 \quad \text{or} \quad Q_2 = \frac{1}{2}P_2$$

So the benchmark is

$$BM_2 = P_2 Q_2 - C_2(Q_1, Q_2)$$

$$= P_2 Q_2 - Q_2^2 - 4$$

$$= \frac{1}{4} P_2^2 - 4$$

7
No matter what Firm 1 does, Firm 2 will maximize profit by setting price equal to marginal cost

$$P_2 = MC_2(Q_1, Q_2) = 2Q_2 + Q_1$$

or

$$Q_2^* = \frac{1}{2}[P_2 - Q_1]$$

or, to emphasize the consequences of Firm 1's actions

$$Q_2^*(Q_1)$$

8

Firm 1 must compensate Firm 2 by the amount

$$BM_2 - \Pi_2 = BM_2 - P_2 Q_2^*(Q_1) + C_2(Q_1, Q_2^*(Q_1))$$

One can show in general that

$$\Delta [BM_2 - \Pi_2] / \Delta Q_1 = MD_2(Q_1, Q_2^*)$$

we shall show it for our specific
example

$$BM_2 - \Pi_2 = BM_2 - P_2 \left(\frac{1}{2}\right) [P_2 - Q_1] + \frac{1}{4} [P_2 - Q_1]^2 + \frac{1}{2} Q_1 [P_2 - Q_1] + 4$$

$$\Delta [BM_2 - \Pi_2] \neq \Delta Q_1 = 0 + \frac{1}{2} P_2 + \frac{1}{2} [P_2 - Q_1] (-1) + \frac{1}{2} P_2 - Q_1 + 0$$

$$= \frac{1}{2} [P_2 - Q_1] - \frac{1}{2} [P_2 - Q_1] + \frac{1}{2} [P_2 - Q_1]$$

$$= Q_2^*$$

$$= MD_2(Q_1, Q_2^*)$$

^ Firm 1's cost function is

$$C_1(Q_1) + [BM_2 - \bar{\Pi}_2]$$

^ Firm 1's marginal cost function is

$$MC_1(Q_1) + MD_2(Q_1, Q_2^*)$$

^ Firm 1 equates price to marginal cost

$$P_1 = MC_1(Q_1) + MD_2(Q_1, Q_2^*)$$

To summarize,

$$P_2 = MC_2(Q_1, Q_2)$$

$$P_1 = MC_1(Q_1) + MP_2(Q_1, Q_2)$$

which are the same equations that describe what should be produced !!

12
What will be profits if firm 1 must pay
Firm 2 and $P_1 = 11$, $P_2 = 10$

Without compensation, from before, (merged firms)

$$Q_1 = 4 \quad \pi_1 = 20$$

$$Q_2 = 3 \quad \pi_2 = 5$$

The benchmark is $BM_2 = \frac{1}{4}P_2^2 - 4 = 21$

Compensation is $21 - 5 = 16$

After compensation

$$Q_1 = 4$$

$$\pi_1 = 4$$

$$Q_2 = 3$$

$$\pi_2 = 21$$