Setup

$P = 60$

$C_1(Q_1) = 300 + 2Q_1^2$

$C_2(Q_2) = 500 + Q_2^2$

$E_1 = Q_1$

$E_2 = Q_2$

$MC_1(Q_1) = 4Q_1$

$MC_2(Q_2) = 2Q_2$

$D(E) = 12Q$

$MD(E) = 12$

What will firms produce

Firm 1

$$\max_{Q_1} \; PQ_1 - C_1(Q_1)$$

$$P = MC_1(Q_1)$$

$$60 = 4Q_1$$

$$Q_1 = 15$$

$$\Pi_1 = 60 \cdot 15 - 300 - 2 \cdot 15^2$$

$$= 150$$

Firm 2

$$\max_{Q_2} \; PQ_2 - C_2(Q_2)$$

$$P = MC_2(Q_2)$$

$$60 = 2Q_2$$

$$Q_2 = 30$$

$$\Pi_2 = 60 \times 30 - 500 - 30^2$$

$$= 400$$

$$E_1 + E_2 = 45$$

$$D(E_1 + E_2) = 45 \cdot 12 = 540$$

What is the efficient outcome

$$\max_{Q_1, Q_2} \; PQ_1 - C_1(Q_1) - PQ_2 - C_2(Q_2) - D(Q_1) - D(Q_2)$$

Q_1: $P - MC_1(Q_1) - MD(Q_1) = 0$

Q_2: $P - MC_2(Q_2) - MD(Q_2) = 0$

$$60 - 4Q_1 - 12 = 0 \quad \Rightarrow \quad Q_1 = 12$$

$$60 - 2Q_2 - 12 = 0 \quad \Rightarrow \quad Q_2 = 24$$

$$\Pi_1 = 60 \cdot 12 - 300 - 2(12)^2 = 132$$

$$\Pi_2 = 60 \cdot 24 - 500 - (24)^2 = 364$$

$$E_1 + E_2 = 36$$

$$D(E_1 + E_2) = 36 \cdot 12 = 432$$

What if government imposes a tax of $12 per unit of emissions?

Firm 1

$$\max_{Q_1} \; PQ_1 - C_1(Q_1) - 12Q_1$$

$$P = MC_1(Q_1) + 12$$

$$60 = 4Q_1 + 12$$

$$Q_1 = 12$$

$$\Pi_1 = 60 \cdot 12 - 300 - 2(12)^2 - 12 \cdot 12$$

$$= -12$$ ← Does not produce

Firm 2

$$\max_{Q_2} \; PQ_2 - C_2(Q_2) - 12Q_2$$

$$P = MC_2(Q_2) + 12$$

$$60 = 2Q_2 + 12$$

$$Q_2 = 24$$

$$\Pi_2 = 60 \cdot 24 - 300 - 24^2 - 12 \cdot 24$$

$$= 76$$

$$E_1 + E_2 = 0 + 24 = 24$$

$$D(E_1 + E_2) = 12 \cdot 24 = 288$$

What if government offers a subsidy of +12 per unit of abatement

Firm 1

Benchmark $Q_1^B = 15$

$$\max_{Q_1} \; PQ_1 - C_1(Q_1) + 12(Q_1^B - Q_1)$$

$$P - MC_1(Q_1) - 12 = 0$$

$$Q_1 = 12$$

$$\Pi_1 = 60 \cdot 12 - 300 - 2(12)^2 + 12(15-12)$$

$$= 168$$

Firm 2

Benchmark $Q_2^B = 30$

$$\max_{Q_2} \; PQ_2 - C_2(Q_2) + 12(Q_2^B - Q_2)$$

$$P - MC_2(Q_2) - 12 = 0$$

$$Q_2 = 24$$

$$\Pi_2 = 60 \cdot 24 - 500 - (24)^2 + (12)(30-24)$$

$$= 436$$

One could make the benchmark the efficient quantities 12 & 24 in which case $Q_1 = 12$, $Q_2 = 24$, $\Pi_1 = 132$, $\Pi_2 = 364$

What if government issues 36 emission permits by auction

Ascending clock: increase price until quantity demanded is equal to supply

↑ firm 1

Willingness to pay for Q_1 permits

$\Pi_1(Q_1) = PQ_1 - C_1(Q_1)$

$MWTP_1 = P - MC_1(Q_1)$

$= 60 - 4Q_1$

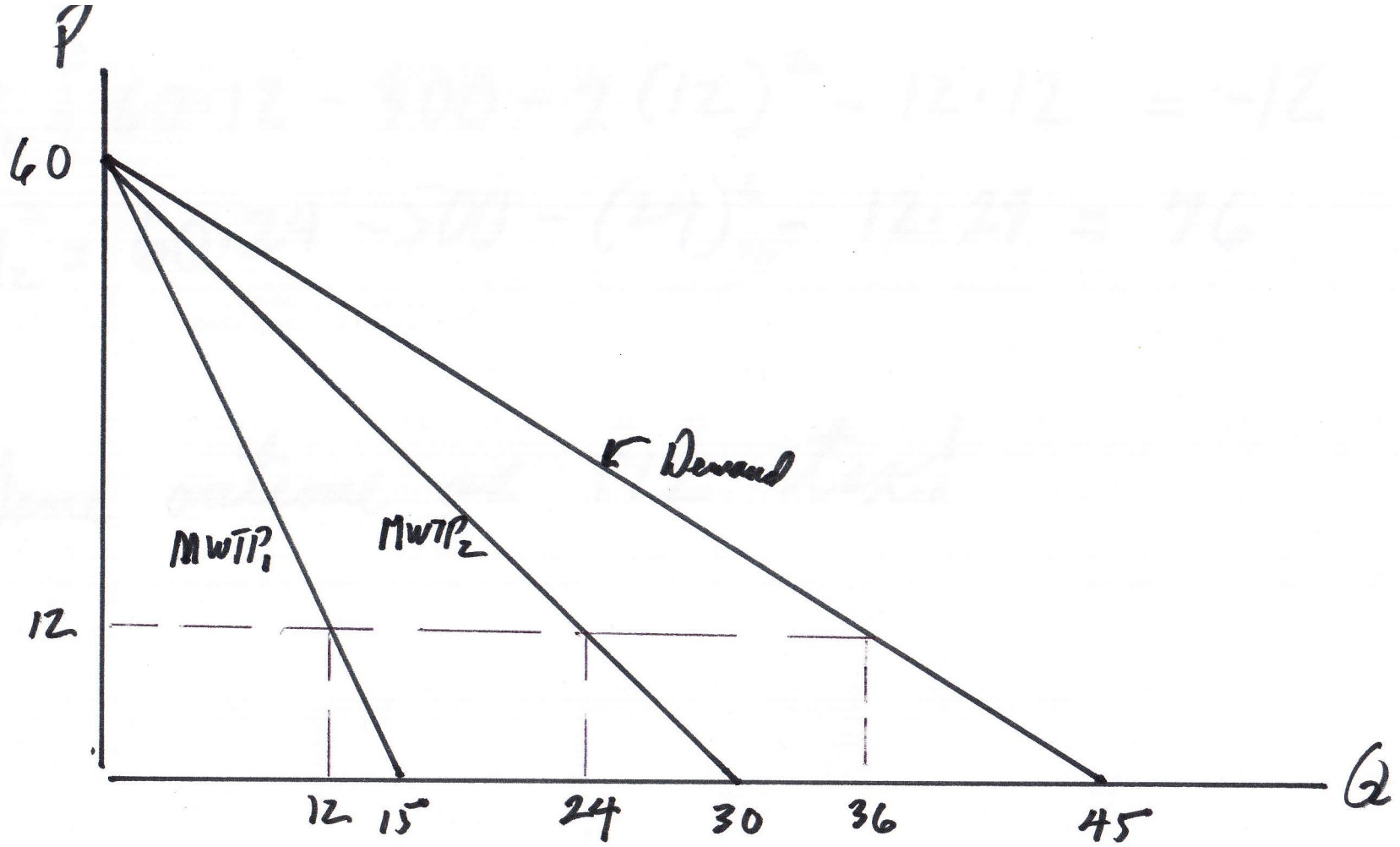
↑ firm 2

Willingness to pay for Q_2 permits

$\Pi_2(Q_2) = PQ_2 - C_2(Q_2)$

$MWTP_2 = P - MC_2(Q_2)$

$= 60 - 2Q_2$



$$\left.\begin{aligned} 60-4Q_1 &= 60-2Q_2 \\ Q_1+Q_2 &= 36 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} 2Q_1 &= Q_2 \\ Q_1+Q_2 &= 36 \end{aligned}\right\} \Rightarrow \begin{aligned} Q_1 &= 12 \\ Q_2 &= 24 \end{aligned}$$

$$MWTP = 60-48 = 12$$

Alternate solution method

$$
\begin{aligned}
Q_1 &= \frac{1}{4}(60 - P) \\
Q_2 &= \frac{2}{4}(60 - P) \\
Q_1 + Q_2 &= \frac{3}{4}(60 - P) \\
36 &= \frac{3}{4}(60 - P) \\
P &= 12 \\
Q_1 &= \frac{1}{4}(60 - 12) = 12 \\
Q_1 &= \frac{2}{4}(60 - 12) = 24
\end{aligned}
$$

$$\Pi_1 = 60 \cdot 12 - 300 - 2(12)^2 - 12 \cdot 12 = -12$$

$$\Pi_2 = 60 \cdot 24 - 500 - (24)^2 - 12 \cdot 24 = 76$$

Same outcome as $12 tax!

What if government gives each firm 18 permits

Firm 1 can only use 15

Firm 2 will use all 18 and could use as many as 30

How much would firm 2 have to pay firm 1 for 6 permits?

$\Pi_1(Q_1=15) - \Pi_1(Q_1=12) = 150 - 132 = 18$

How much would firm 2 pay for 6 permits?

$\Pi_2(Q_2=24) - \Pi_2(Q_2=18) = 364 - 256 = 108$

There are gains to trade because

$18 \leq \text{X} \leq 108$

$P_X X$

If firm 1 has 12 permits and firm 2 has 24 permits, there are no gains from trade

Marginal value to 1 of one more $P - MC_1(Q_1 = 12) = 12$

Marginal value to 2 of one more $P - MC_2(Q_2 = 12) = 12$

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