The Impact of Economic and Climate Risks on the Social Cost of Carbon

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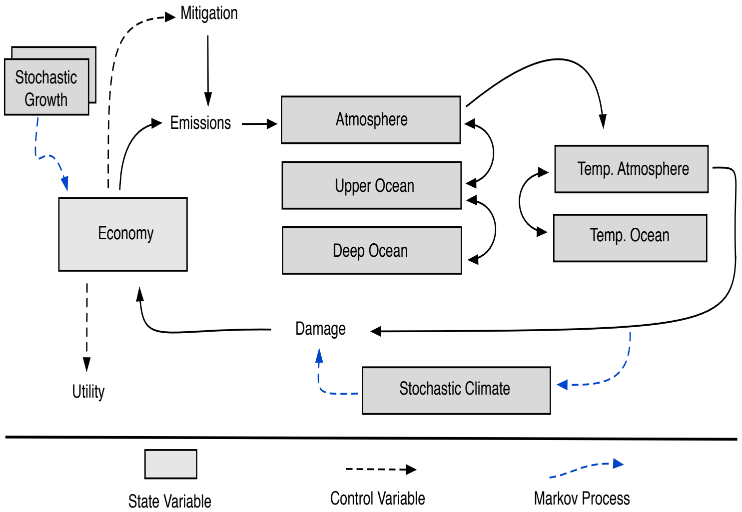
Climate Change Policy Analysis

Question: What can and should be the policy response to rising CO2 concentrations in the face of uncertainty?

- Create dynamic and stochastic integrated models of climate and economy (DSICE)
  - Economic risk:
    - uncertain economic growth with persistence in growth rates, calibrated to consumption data
    - flexible preferences compatible with data on asset pricing: Epstein–Zin preferences
  - Climate risk
    - damages interact with economic shocks
    - climate events are stochastic; e.g., glaciers melting, THC collapse
  - Parameter uncertainty
- Results
  - SCC (Social Cost of Carbon) today is higher, ~double the $35/tC "consensus"
  - SCC is a stochastic process:
    - policies aimed at reducing emissions (e.g., carbon tax) could hit their maximum effectiveness in this century
    - carbon sequestration and geoengineering may be cost-effective

DSICE Framework

DSICE: Dynamic Stochastic Integration of Climate and the Economy
Extension of Nordhaus' DICE to economic and climate riskiness



Economic System in DSICE

- Production function without climate effects:

$$f(K_t, L_t, \widetilde{A}_t) = \widetilde{A}_t K_t^{\alpha} L_t^{1-\alpha}$$

  - K_t: capital;
  - L_t: world population
  - $\widetilde{A}_t$: stochastic productivity, $\widetilde{A}_t \equiv \zeta_t A_t$
    - A_t: deterministic trend
    - ζ_t: productivity shock with long-run risk

$$\log(\zeta_{t+1}) = \log(\zeta_t) + \chi_t + \varrho\omega_{\zeta,t}$$

$$\chi_{t+1} = r\chi_t + \varsigma\omega_{\chi,t}$$

- Output:

$$\mathcal{Y}_t = \Omega\left(T_{\mathrm{AT},t}, J_t\right) f\left(K_t, L_t, \zeta_t A_t\right)$$

  - $T_{\mathrm{AT},t}$: atmospheric temperature; Ω: damage factor
  - J_t: climate state

Economic System in DSICE

- Capital accumulation:

$$K_{t+1} = (1-\delta)K_t + \mathcal{Y}_t - C_t - \Psi_t \tag{1}$$

  - C_t: consumption;
  - μ_t: emission control rate
  - Ψ_t: mitigation expenditure, $\Psi_t = \theta_{1,t}\mu_t^{\theta_2}\mathcal{Y}_t$
- Epstein–Zin Preferences:
  - ψ: inter temporal elasticity of substitution
  - γ: risk aversion parameter
  - parameters chosen to imply plausible risk premia in asset markets and IES for consumption

Climate System in DSICE – Heat and GHG Diffusions

- Carbon concentration: $\mathbf{M} = (M_{\mathrm{AT}}, M_{\mathrm{UO}}, M_{\mathrm{LO}})$

$$\mathbf{M}_{t+1} = \Phi_M \mathbf{M}_t + (\mathcal{E}_t, 0, 0)^\top$$

  - $\mathcal{E}_t$: emission depending on production and emission control rate μ_t
  - Φ_M: transition matrix of carbon cycle
- Temperature: $\mathbf{T} = (T_{\mathrm{AT}}, T_{\mathrm{OC}})$

$$\mathbf{T}_{t+1} = \Phi_T \mathbf{T}_t + (\xi_1 \mathcal{F}_t (M_{\mathrm{AT},t}), 0)^\top \tag{2}$$

  - $\mathcal{F}_t$: radiative forcing
  - Φ_T: transition matrix of temperature system
- Tipping Element: J_t
- Damage in output:

$$\Omega (T_{\mathrm{AT},t}, J_t) = \frac{1 - J_t}{1 + \pi_1 T_{\mathrm{AT},t} + \pi_2 (T_{\mathrm{AT},t})^2}$$

Climate System in DSICE – Accumulated Damage State

- Climate state: J_t
  - Represents past, permanent harm
  - Markov chain, transition probabilities depends on the contemporaneous temperature T_{AT}
  - multi-stage process of uncertain duration
- Examples of tipping elements
  - ice sheet melting (West Antarctic, Greenland)
  - collapse of Atlantic themohaline circulation

Bellman Equation

- Nine-dimensional state vector: $S = (K, \mathbf{M}, \mathbf{T}, \zeta, \chi, J)$
- Bellman equation for the dynamic stochastic problem:

$$
\begin{aligned}
V_t(S) = \max_{C,\mu} \quad & u_t(C_t, L_t) + \beta \left[\mathbb{E}_t \left\{ \left(V_{t+1}\left(S^+\right)\right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \right\} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}, \\
\text{s.t.} \quad & K^+ = (1-\delta)K_t + \mathcal{Y}_t - C_t - \Psi_t, \\
& \mathbf{M}^+ = \Phi_M \mathbf{M} + (\mathcal{E}_t, 0, 0)^\top, \\
& \mathbf{T}^+ = \Phi_T \mathbf{T} + \left(\xi_1 \mathcal{F}_t\left(M^{AT}\right), 0\right)^\top, \\
& \zeta^+ = g_\zeta(\zeta, \chi, \omega_\zeta), \\
& \chi^+ = g_\chi(\chi, \omega_\chi), \\
& J^+ = g_J(J, \mathbf{T}, \omega_J)
\end{aligned}
$$

- 600 years horizon in annual time steps

Social Cost of Carbon and Carbon Tax

- SCC (the 1000 factor corrects for difference in units):

$$\varGamma_t = -1000\left(\partial V_t / \partial M_{\mathrm{AT},t}\right) / \left(\partial V_t / \partial K_t\right). \tag{3}$$

- Relation of SCC and carbon tax:
  - if $\mu_t < 1$, SCC = Carbon tax
  - if $\mu_t = 1$ (i.e., no industrial emission), SCC > Carbon tax, implying that mitigation policies reach their limit of effectiveness
- When SCC is high, alternative policies may be efficient (e.g., carbon removal and storage, solar geoengineering)

Calibration - Benchmark Case

- Preferences: $\psi = 1.5$, $\gamma = 10$ (plausible case)
- Productivity: Calibrate three parameters (ϱ, r, ς) in exogenous stochastic productivity process:
  - Solve DSICE and compute consumption assuming no climate damages
  - Compare the moments of per-capita consumption growth with empirical data:
- Climate tipping processes: Use expert elicitation studies (Kriegler et al. 2009; Lenton 2010)

Numerical Dynamic Programming

- DSICE:
  - six-dimensional continuous state variables $x \equiv (K, \mathbf{M}, \mathbf{T})$
  - three discrete state variables $\theta \equiv (\zeta, \chi, J)$ with $91 \times 19 \times 16$ time-dependent values
- Numerical Dynamic Programming Algorithm:
  - *Initialization. Choose the approximation nodes,* $\mathbb{X}_t = \{x_{i,t} : 1 \leq i \leq m_t\}$ *for every* $t < \mathcal{T}$*, and choose a functional form for* $\hat{V}(x, \theta; \mathbf{b})$*, where* $\theta \in \Theta_t$*. Let* $\hat{V}(x, \theta; \mathbf{b}_{\mathcal{T}}) \equiv V_{\mathcal{T}}(x, \theta)$*. Then for* $t = \mathcal{T} - 1, \mathcal{T} - 2, \ldots, 0$*, iterate through steps 1 and 2.*
  - *Step* ***1.*** *Maximization step. Compute*

$$
\begin{aligned}
v_{i,j} = \max_{a \in \mathcal{D}(x_i, \theta_j, t)} \quad & u_t(x_i, a) + \beta \mathcal{H}_t \left(\hat{V}\left(x^+, \theta_j^+; \mathbf{b}_{t+1}\right)\right) \\
\text{s.t.} \quad & x^+ = F(x_i, \theta_j, a), \\
& \theta_j^+ = G(x_i, \theta_j, \omega),
\end{aligned}
$$

    for each $\theta_j \in \Theta_t$*,* $x_i \in \mathbb{X}_t$*,* $1 \leq i \leq m_t$*.*
  - *Step* ***2.*** *Fitting step. Using an appropriate approximation method, compute the* $\mathbf{b}_t$ *such that* $\hat{V}(x, \theta_j; \mathbf{b}_t)$ *approximates* $(x_i, v_{i,j})$ *data for each* $\theta_j \in \Theta_t$*.*

LRR Benchmark - GWP, K, C

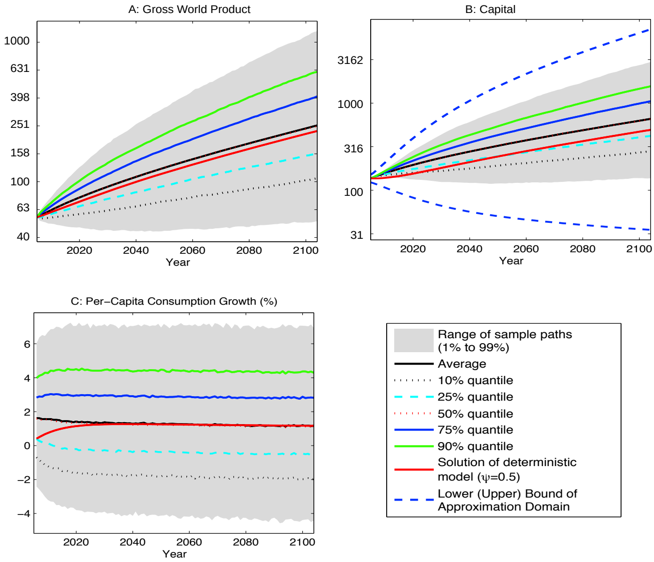


Figure : Simulation results of the stochastic growth benchmark

LRR Benchmark – Emission Control, Carbon, Temperature

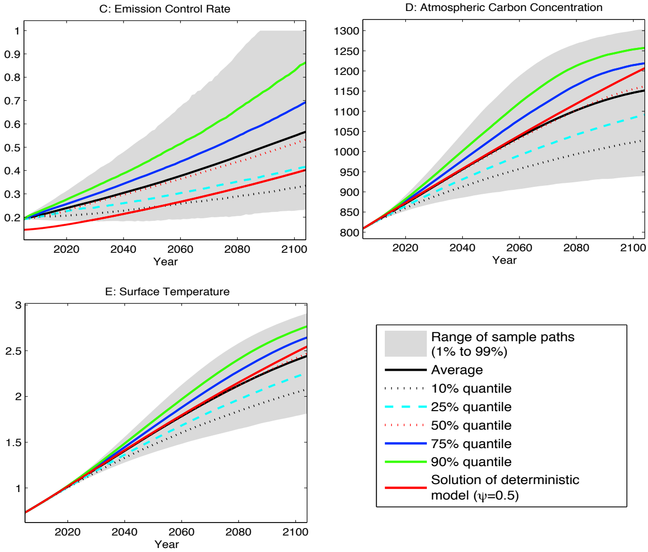
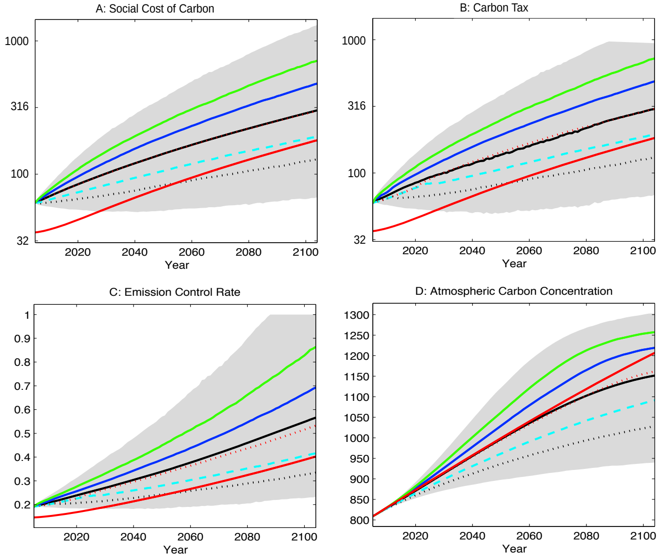


Figure : Simulation results of the stochastic growth benchmark

SCC and Carbon Tax



- Optimal Initial carbon tax: \$125 (deterministic model: \$37)

Parallelization of DSICE – LRR+Tipping

- Discretized dimensions (ζ, χ, J): $91 \times 19 \times 16 = 27,664$ points
- Six-dimensional continuous states $(k, \mathbf{M}, \mathbf{T})$:
  - 56K approximation nodes per discrete point
  - 261 coefficients in polynomial approximation at each discrete (ζ, χ, J) point
  - massive overidentification is needed to get good approximation of value function and the decision rules (which are essentially the gradients)
- Value function iteration method
- Total number of Bellman optimization problems: 372 billion

Num of Cores	Wall Clock Time	Total CPU Time
69,184	11.2 hours	88 years

Parallelization of Uncertainty Quantification – Tipping

- Six uncertain parameter values
  - intertemporal elasticity of substitution
  - risk aversion parameter
  - hazard rate of tipping
  - expected damages
  - variance of damages
  - expected duration of the tipping process
- Solve on grids in parameter space (2,430 cases)
- Use approximation to express SCC now as a function of six parameters to at least three-digit accuracy
- Computational resources.

Num of Cores	Wall Clock Time	Total CPU Time
8,160	1.04 hour	0.97 year

Summary

- We construct a DSICE model with economic and climate uncertainty
- We use good mathematical methods; will use better in the future
- We use standard scientific computer hardware
- DSICE shows that mild tipping specifications can lead to very high SCC; disaster scenarios are not needed to justify high SCC
- DSICE shows that parameter uncertainty implies even more uncertainty
- DSICE shows that SCC is a stochastic process with wide ranges over time
    - It is quite plausible that in this century optimal policy would not only eliminate CO2 emissions,
    - but also employ far more costly measures aimed at removing CO2 from the atmosphere
    - DSICE could be used for cost-benefit analysis of scientific studies reducing uncertainty, carbon removal and storage, geoengineering, and other possibilities