UNIVERSITY OF NORTH CAROLINA
Department of Economics

Economics 271 | Dr. Gallant
Midterm Exam | Fall 2000
Oct. 11, 2000

1. (20%) Complete the following table

$f(x,y)$	y						
x	1	2	3	4	5	$f(x)$	$\mathcal{E}(Y\|X)(x)$
1	.01	.02	.03	.03	.04		
2	.03	.03	.04	.05	.05		
3	.03	.05	.04	.05	.06		
4	.02	.02	.04	.04	.05		
5	.04	.05	.05	.06	.07		

2. (20%) Let $(\Omega, \mathcal{F}, P)$ be the coin tossing probability space; i.e., $\Omega = (0,1]$, $\mathcal{F}$ is the smallest σ-algebra that contains all finite unions of sets of the form $(a,b]$, and $P(a,b] = b - a$. Consider the random variable $X(\omega) = \sqrt{\omega}$, which maps $(\Omega, \mathcal{F}, P)$ into a new probability space $(\mathcal{X}, \mathcal{A}, P_X)$.

   (a) What is $\mathcal{X}$ in this new probability space?

   (b) What is $\mathcal{A}$ in this new probability space?

   (c) If $(c,d] \subset \mathcal{X}$, what is the value of $P_X(c,d]$?

   (d) What is the density $f_X(x)$ of the random variable X?

   (e) What is the value of $\mathcal{E}X$?

3. (20%) A pair of dice are thrown and the value $\omega = (n_1, n_2)$ is observed where n_1 is the number of spots showing on the first die and n_2 is the number of spots showing on the second.

   (a) What is the joint probability density function $f_{X,Y}(x,y)$ of the random variables $X(\omega) = (n_1 - n_2)$, $Y(\omega) = (n_1 + n_2)$?

(b) What is the value of $\mathcal{E}X$?

(c) Let $F = \{\omega : X(\omega) = 1\}$. What is the value of $\mathcal{E}(Y|F)$?

4. (20%) For each of the following, if the statement is true, then prove it, if the statement is false, then give a counter example.

   (a) The collection $\mathcal{A} = \{\emptyset, \Omega, A, \tilde{A}\}$ a σ-algebra.

   (b) The union of two σ-algebras a σ-algebra.

   (c) The intersection of two σ-algebras a σ-algebra.

5. (20%) Assume that $0 \leq X(\omega) \leq Y(\omega)$. Use the definition of expectation

$$\mathcal{E}X = \sup\left\{\mathcal{E}X_N : X_N(\omega) = \sum_{i=1}^{N} x_i I_{F_i}(\omega), 0 \leq X_N(\omega) \leq X(\omega)\right\}$$

to show directly that $0 \leq \mathcal{E}X \leq \mathcal{E}Y$.