UNIVERSITY OF NORTH CAROLINA
Department of Economics

Economics 271
Midterm Exam
Oct. 15, 2000

Dr. Gallant
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1. (25%) The coin tossing probability space is $(\Omega, \mathcal{F}, P)$, where $\Omega = (0,1]$, $\mathcal{F}$ is the smallest σ-algebra containing all intervals of the form $(a,b]$, $0 \le a \le b \le 1$, and $P(A) = \int I_A(\omega)\,d\omega$. Let $F_1 = (0,\,1/2]$, $F_2 = (0,\,1/4] \cup (1/2,\,3/4]$, $X_1(\omega) = I_{F_1}(\omega)$, $X_2(\omega) = I_{F_2}(\omega)$ and $Y = X_1 + X_2$.

   (a) Show that F_1 and F_2 are independent events.

   (b) Derive the densities $f_{X_1}(x_1)$, $f_{X_2}(x_2)$, $f_{X_1,X_2}(x_1,x_2)$ of X_1, X_2, and (X_1,X_2), respectively.

   (c) Show that $f_{X_1,X_2}(x_1,x_2) = f_{X_1}(x_1) f_{X_2}(x_2)$

   (d) F_1 is the event "tails on the first toss", F_2 is the event "tails on the second toss." Write down the event "tails on the third toss" in terms of unions of intervals of the form $(a,\,b]$.

   (e) Is $F_1 \cup F_2$ the event "one tail in the first two tosses" or is it the event "at least one tail on the first two tosses?"

   (f) Derive the density $f_Y(y)$ of Y.

2. (20%) Complete the following table

$f(x,y)$	y						
x	1	2	3	4	5	$f(x)$	$\mathcal{E}(Y \mid X)(x)$
1	.01	.02	.03	.03	.04		
2	.04	.05	.04	.05	.06		
3	.03	.03	.05	.05	.05		
4	.02	.02	.03	.04	.05		
5	.04	.05	.05	.06	.06		

3. (20%) A pair of dice are thrown and the sum is noted. The throws are repeated until either a sum of 6 or a sum of 7 occurs.

   (a) What is the sample space for this experiment.

   (b) What is the probability that the sequence of throws terminates in a 7.

4. (15%) Show that the intersection of two σ-algebras is a σ-algebra.

5. (20%) If A and B are subsets of $\mathcal{X}$, and $A_1, A_2, \ldots$ is a sequence of subsets from $\mathcal{X}$, show that the inverse image satisfies these properties:

   (5) $X^{-1}\left(\bigcap_{i=1}^{\infty} A_i\right) = \bigcap_{i=1}^{\infty} X^{-1}\left(A_i\right)$

   (6) If $h(\omega) = g[X(\omega)]$, then $h^{-1}(B) = X^{-1}[g^{-1}(B)]$

   You may use these facts without proof in your answer:

   (1) If $A \subset B$, then $X^{-1}(A) \subset X^{-1}(B)$

   (2) $X^{-1}(A \cup B) = X^{-1}(A) \cup X^{-1}(B)$

   (3) $X^{-1}(A \cap B) = X^{-1}(A) \cap X^{-1}(B)$

   (4) $X^{-1}\left(\bigcup_{i=1}^{\infty} A_i\right) = \bigcup_{i=1}^{\infty} X^{-1}\left(A_i\right)$

   (7) $X^{-1}(\sim A) = \sim X^{-1}(A)$