UNIVERSITY OF NORTH CAROLINA
Department of Economics

Economics 271 | Dr. Gallant
Midterm Exam | Fall 2002
Oct. 14, 2002

1. (30%) The coin tossing probability space is $(\Omega, \mathcal{F}, P)$, where $\Omega = (0, 1]$, $\mathcal{F}$ is the smallest σ-algebra containing all intervals of the form $(a, b]$, where $0 \le a \le b \le 1$, and $P(F) = \int I_F(\omega)\, d\omega$. Consider the following events

$F_1 = (\frac{1}{2},\, 1]$	heads on the first toss
$F_2 = (\frac{1}{4},\, \frac{1}{2}] \cup (\frac{3}{4},\, 1]$	heads on the second toss
$F_3 = (\frac{1}{8},\, \frac{1}{4}] \cup (\frac{3}{8},\, \frac{1}{2}] \cup (\frac{5}{8},\, \frac{3}{4}] \cup (\frac{7}{8},\, 1]$	heads on the third toss

Let $X(\omega) = \frac{1}{3}\omega^3$.

(a) Show that F_1 and F_2 are independent events.

(b) Show that F_1, F_2, and F_3 are independent events.

(c) Derive the density function $f_X(x)$ of X.

(d) Derive the distribution function $F_X(x)$ of X.

2. (20%) Complete the following table

$f(x,y)$	y							
x	1	2	3	4	5	$f(x)$	$F(x)$	$\mathcal{E}(Y\|X)(x)$
1	.02	.02	.03	.05	.04			
2	.05	.04	.04	.04	.05			
3	.02	.01	.03	.06	.03			
4	.03	.03	.06	.05	.05			
5	.05	.04	.05	.06	.05			

3. (10%) Show that $I_{X^{-1}(F)}(\omega) = I_F[X(\omega)]$.

4. (25%) Let X and Y be continuous random variables with joint density

$$f_{X,Y}(x,y) = \begin{cases} x+y & 0<x<1,\ 0<y<1 \\ 0 & \text{otherwise} \end{cases}$$

Compute

(a) $f_X(x)$

(b) $\mathcal{E}(X)$

(c) $\mathcal{E}(Y|X)(x)$

(d) $F_{X,Y}(x,y)$

5. (15%) A pair of dice are thrown and the sum is noted. The throws are repeated until either a sum of 4 or a sum of 7 occurs. What is the sample space for this experiment? What is the probability that the sequence of throws terminates in a 7? Be sure to include an explanation of the logic that you used to reach your answer.