UNIVERSITY OF NORTH CAROLINA

Department of Economics

Economics 271 Dr. Gallant

Midterm Exam Fall 1995

(25%) 1. Let $X(s)$ be a random variable mapping the sample space $(\mathcal{S}, \mathcal{B}, P)$ onto $(\mathcal{X}, \mathcal{A}, P_X)$.

a) For $B \in \mathcal{B}$ define the image $X(B)$.

b) For $A \in \mathcal{A}$ define the preimage $X^{-1}(A)$.

c) What is the image of $\mathcal{S}$?

d) What is the preimage of $\mathcal{X}$.

e) What is the definition of P_X.

f) What is the definition of the distribution function F_X.

(15%) 2. Let $X(s)$ be a random variable mapping the sample space $(\mathcal{S}, \mathcal{B}, P)$ onto $(\mathcal{X}, \mathcal{A}, P_X)$. Let $w = W(x)$ be a one-to-one, increasing function mapping $\mathcal{X}$ onto $\mathcal{W}$ with inverse $W^{-1}(w)$. Show that $F_W(t) = F_X[W^{-1}(t)]$.

(25%) 3. Let X be a random variable that maps the sample space $\mathcal{S}$ onto $\mathcal{X}$. Let $\mathcal{A}$ be a σ-algebra of subsets of $\mathcal{X}$. Show that the collection of sets $\mathcal{F}$ that consists of all preimages of sets A from $\mathcal{A}$ is a σ-algebra. That is, show that

$$\mathcal{F} = \{F : F = X^{-1}(A), A \in \mathcal{A}\}$$

is a σ-algebra. Hint: Recall that $\bigcup_{i=k}^{\infty} X^{-1}[A_i] = X^{-1}[\bigcup_{i=k}^{\infty} A_i]$ and that $\widetilde{X^{-1}}[A] = X^{-1}[\tilde{A}]$

(15%) 4. A pair of dice are thrown and the sum is noted. The throws are repeated until either a sum of 5 or a sum of 7 occurs.

a) What is the sample space for this experiment?

b) What is the probability that the sequence of throws terminates in a 7?

Be sure to include an explanation of the logic that you used to reach your answer.

(20%) 5. Compute the first three moments of the normal distribution.