UNIVERSITY OF NORTH CAROLINA
Department of Economics

Economics 271 — Dr. Gallant
Midterm Exam — Fall 1996
Oct. 15, 1996

1. (25%) Shown in the upper panels of Figure 1 is a plot of the domain $\Omega = (0,1) \times (0,1)$ and range $R = (X,Y)(\Omega)$ for the random variables

$$X = \omega_1 + \omega_2$$

$$Y = \omega_1 - \omega_2$$

Find the density $f_{X,Y}(x,y)$ of (X,Y). Find the inverse image of the set A shown in the lower left panel of Figure 1 and draw it in the lower right panel. Compute the probability of A.

2. (15%) A pair of dice are thrown and the sum is noted. The throws are repeated until either a sum of 6 or a sum of 7 occurs. What is the sample space for this experiment? What is the probability that the sequence of throws terminates in a 7? Be sure to include an explanation of the logic that you used to reach your answer.

3. (15%) Show that the intersection of two σ-algebras is a σ-algebra.

4. (15%) In a shipment of 1,000 transistors, 100 are defective. If 50 transistors are inspected, what is the probability that 5 of them will be defective. Be sure to include an explanation of the logic that you used to reach your answer.

5. (15%) Prove that if $P(B) = 1$, then $P(A|B) = P(A)$ for any A. Prove that if $A \subset B$, then $P(B|A) = 1$.

6. (15%) If A and B are subsets of $\mathcal{X}$, and $A_1, A_2, \ldots$ is a sequence of subsets from $\mathcal{X}$, show that the inverse image satisfies these properties: (i) If $A \subset B$, then $X^{-1}(A) \subset X^{-1}(B)$. (vi) If $h(\omega) = g[X(\omega)]$, then $h^{-1}(B) = X^{-1}[g^{-1}(B)]$

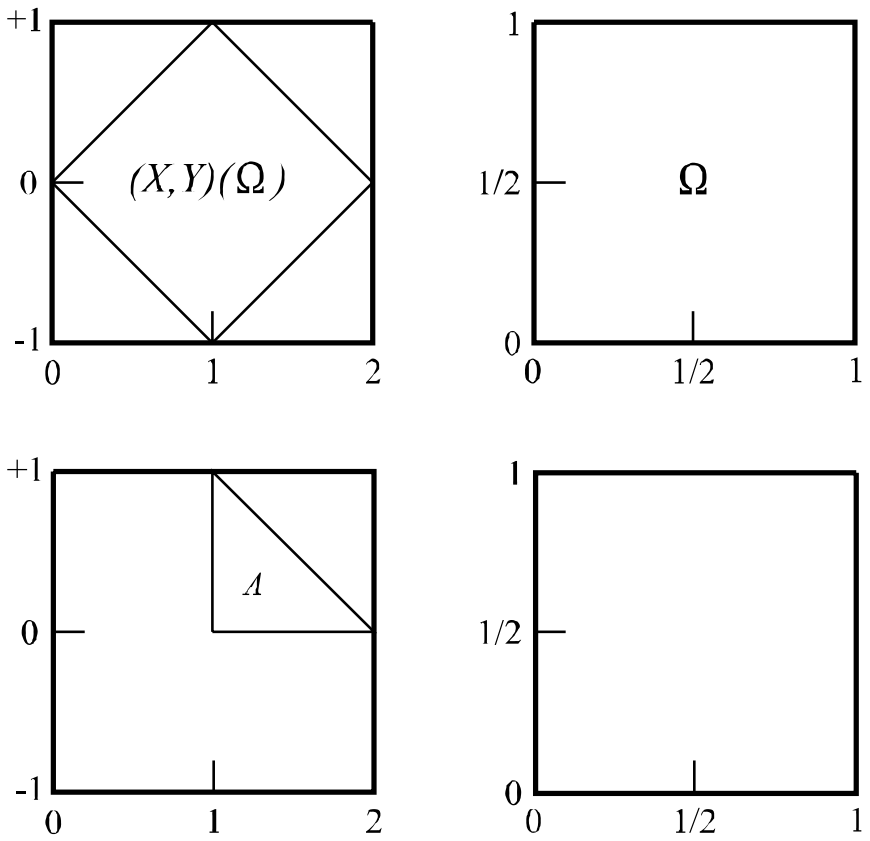

Figure 1. A Bivariate Random Variable Defined on the Double Coin Toss Probability Space. The sample space is $\Omega = (0,1) \times (0,1)$, shown in the upper right panel on which is defined the random variable $(X,Y)(\omega_1,\omega_2) = (\omega_1+\omega_2, \omega_1-\omega_2)$. Its range $R = (X,Y)(\Omega)$ is shown in the upper left panel.