UNIVERSITY OF NORTH CAROLINA
Department of Economics

Economics 271
Midterm Exam
Oct. 4, 1999

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1. (20%) Complete the following table

$f(x,y)$	y						
x	1	2	3	4	5	$f(x)$	$\mathcal{E}(Y\|X)(x)$
1	.01	.02	.03	.03	.04		
2	.02	.03	.03	.04	.05		
3	.03	.03	.04	.05	.05		
4	.03	.04	.05	.05	.06		
5	.04	.05	.05	.06	.07		

2. (15%) Show that

$$I_{X^{-1}(F)}(\omega) = I_F[X(\omega)].$$

3. (30%) A probability space $(\Omega, \mathcal{F}, P)$ satisfies three properties:

   (a) $P(A) \geq 0$ for all $A \in \mathcal{F}$.

   (b) $P(\Omega) = 1$.

   (c) If $A_1, A_2, \ldots \in \mathcal{F}$ are disjoint, then $P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$.

   Use these three properties to show that

   (a) $P(\emptyset) = 0$.

   (b) $P(A) \leq 1$.

   (c) $P(A) + P(\tilde{A}) = 1$.

   (d) $P(A \cap B) + P(A \cap \tilde{B}) = P(A)$.

   (e) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

   (f) If $A \subset B$, then $P(A) \leq P(B)$.

4. (20%) Let the random variable Y be defined by

$$Y = \beta_0 + \beta_1 x + \omega,$$

where ω is gotten by coin tossing and the value of x is known. Find the density $f_Y(y)$ of the random variable Y. What is the value of $\mathcal{E}Y$?

5. (15%) In a shipment of 1,000 transistors, 200 are defective. If 25 transistors are inspected, what is the probability that 5 of them will be defective. Be sure to include an explanation of the logic that you used to reach your answer.