Does Smooth Ambiguity Matter for Asset Pricing?

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Motivation

Existing studies on ambiguity and asset pricing are confined to calibration.

- Multiple priors: Epstein and Wang (1994), Chen and Epstein (2002), Epstein and Miao (2003), Epstein and Schneider (2008), Drechsler (2013)
- Smooth ambiguity: Ju and Miao (2012), Jahan-Parvar and Liu (2014), Collard, Mukerji, Sheppard and Tallon (2016)
- Robustness: Anderson, Hansen and Sargent (2003), Cagetti, Hansen, Sargent and Williams (2002), Hansen (2007), Hansen and Sargent (2001, 2008, 2010)

Motivation

This study: estimating a class of consumption-based asset pricing models with smooth ambiguity (Hayashi and Miao (2011); Klibanoff, Marinacci and Mukerji (2005, 2009))

- Structural estimation
  - joint estimation of consumption dynamics and preference parameters
- Five models
  - Ju and Miao (2012): "Ambiguity, Learning and Asset Returns" (AAMS)
  - Ju&Miao + time-varying volatility (AAMSSV)
  - long-run risk + ambiguity + stochastic volatility (AALRRSV)
  - Bansal, Kiku and Yaron (2012): long-run risk + stochastic vol (EZLRRSV)
  - Markov-switching consumption growth + learning (EZMS)

Motivation

Questions to ask:

- Any statistical support for asset pricing models with smooth ambiguity?
- Does structural estimation lend support to models with time-varying volatility v.s. Ju and Miao's model?
- Model comparison: smooth ambiguity or long-run risks?
    - Bansal, Gallant and Tauchen (2007) and Aldrich and Gallant (2011): the LRR model is a preferred model (compared to habit formation, prospect theory)
    - Schorfheide, Song and Yaron (2017): Bayesian estimation of LRR models

Motivation

Main difficulties

- Likelihood of an asset pricing model is unknown
  - we use projection methods to solve models — free from bias due to log-linearization (Pohl, Schmedders and Wilms, 2017)
  - cost: likelihood is not readily available
- Sparse data
  - priors on structural parameters may be helpful

Motivation

Estimation method: Bayesian estimation of "General Scientific Models (GSM)", Gallant and McCulloch (2008) and Aldrich and Gallant (2011)

- Bayesian indirect inference
- synthesize a likelihood by means of an auxiliary model and simulations from a scientific (asset pricing) model.
- the auxiliary model (*sieve*)
  - extend Aldrich&Gallant&McCulloch's auxiliary model to a four-dimensional model
  - semi-nonparametric (SNP) estimation, Gallant and Tauchen (1989, 2010)
    1. the leading term of the (Hermite) series expansion: an established parametric model
    2. higher order terms: capture departures from that model.

Bayesian Estimation of GSM: Basics

- Let the transition density of the structural model be:

$$p(y_t|z_{t-1},\theta), \quad \theta \in \Theta,$$

where $z_{t-1} = (y_{t-1}, ..., y_{t-L})$ and Θ is the set of structural parameters in an asset pricing model.

- Assume that there is a transition density (auxiliary model)

$$f(y_t|z_{t-1},\omega), \quad \omega \in \Omega$$

where Ω is the set of parameters in the auxiliary model

- Assume there is an map between the structural model parameters and the auxiliary model parameters:

$$g : \theta \mapsto \omega$$

such that $p(y_t|z_{t-1},\theta) = f(y_t|z_{t-1},g(\theta)), \qquad \theta \in \Theta.$

Bayesian Estimation of GSM: Roadmap

1. Find an auxiliary model $f(y_t|z_{t-1},\omega)$ that fits the observed data $\{\tilde{y}_t\}_{t=1}^{n}$ well.

2. Find the implied map $g:\theta \mapsto \omega$ that satisfies

$$p(y_t|z_{t-1},\theta) = f(y_t|z_{t-1},g(\theta))$$

3. Then the likelihood of the observed data given the structural parameters θ can be described as

$$\mathcal{L}\left(g\left(\theta\right)\right) = \prod_{t=1}^{n} f(y_t|z_{t-1},g(\theta))$$

Bayesian Estimation of GSM: the Implied Map

- Suppose that we have simulated a long time series data from the structural model, $\{\hat{y}_t\}_{t=1}^{N}$ given a set of structural parameters θ.
- How to find the implied map $g : \theta \mapsto \omega$?
- Given θ, compute $\omega = g(\theta)$ by minimizing Kullback-Leibler divergence w.r.t. ω:

$$KL(f,p) = \int\int \left[\ln p(y|z,\theta) - \ln f(y|z,\omega)\right] p(y|z,\theta)\, dy\, p(z|\theta)\, dz$$

Bayesian Estimation of GSM: the Implied Map

- Minimizing the K-L divergence is equivalent to finding the map by

$$g : \theta \mapsto \arg\max_{\omega} \sum_{t=1}^{N} \ln f(\hat{y}_t \mid \hat{z}_{t-1}, \omega)$$

— compute the MLE of ω for the simulated data $\{\hat{y}_t\}_{t=1}^{N}$.

- Compute the maximizing value $\hat{\omega}$ using a Metropolis-Hastings chain — computationally costly

- Gallant and McCulloch's GSM package: BFGS quasi-Newton method — helps shorten the chain

Bayesian Estimation of GSM: Metropolis-Hastings

$\mathcal{L}(\theta)$: the likelihood of the observed data (y_t, z_{t-1}) with sample size n assuming that the map $g : \theta \mapsto \omega$ exists.

$$\mathcal{L}(\theta) = \prod_{t=1}^{n} f\left(y_t | z_{t-1}, g(\theta)\right)$$

- Let $\xi(\theta)$ denote the prior distribution on θ.
  - We use loose priors — data has influential effect
  - Tight enough — MCMC chains can mix well

- Let q denote a proposal density. For a given θ, $q(\theta, \theta^*)$ defines a distribution of potential new values θ^*.
  - move-one-at-a-time, random-walk proposal density

Bayesian Estimation of GSM: Metropolis-Hastings

Given a current θ^o and the corresponding $\omega^o = g(\theta^o)$, we obtain the next pair (θ', ω') as follows:

1. Draw θ^* according to $q\left(\theta^o, \theta^*\right)$
2. Draw $\{\hat{y}_t, \hat{z}_{t-1}\}_{t=1}^N$ according to $p\left(y_t | z_{t-1}, \theta^*\right)$
3. Compute $\omega^* = g\left(\theta^*\right)$ from the simulation $\{\hat{y}_t, \hat{z}_{t-1}\}_{t=1}^N$
4. Let
$$\alpha = \min\left(1, \frac{\mathcal{L}\left(\theta^*\right)\xi\left(\theta^*\right)q\left(\theta^*, \theta^o\right)}{\mathcal{L}\left(\theta^o\right)\xi\left(\theta^o\right)q\left(\theta^o, \theta^*\right)}\right)$$
5. With probability α, set $(\theta', \omega') = (\theta^*, \omega^*)$, otherwise set $(\theta', \omega') = \left(\theta^{^o}, \omega^o\right)$.

The Auxiliary Model: SNP estimation

A modified multivariate GARCH

$$y_t = \mu_{z_{t-1}} + U_{z_{t-1}} \varepsilon_t$$

$$\mu_{z_{t-1}} = b_0 + B z_{t-1},$$

$U_{z_{t-1}}$ is the Cholesky factor of

$$\begin{aligned} \Sigma_{z_{t-1}} = \; & U_0 U_0' \\ & + Q \Sigma_{z_{t-2}} Q' \\ & + P(y_{t-1} - \mu_{z_{t-2}})(y_{t-1} - \mu_{z_{t-2}})' P' \\ & + \max[0, \tilde{V}(y_{t-1} - \mu_{z_{t-2}})] \max[0, \tilde{V}(y_{t-1} - \mu_{z_{t-2}})]', \end{aligned}$$

- Density $h(\varepsilon)$ of ε_t: the square of a Hermite polynomial times a normal density,
- Model (lag&polynomial orders) selection: BIC
- 50 parameters in the selected model

Asset Pricing Models

- Ju and Miao (2012): "Ambiguity, Learning and Asset Returns" (AAMS)
- Ju&Miao + time-varying volatility (AAMSSV)
- long-run risk + ambiguity + stochastic volatility (AALRRSV)
- Bansal, Kiku and Yaron (2012): long-run risk + stochastic vol (EZLRRSV)
- Markov-switching consumption growth + learning (EZMS)

AAMS: Ju and Miao (2012)

- Consumption and dividend processes

$$\Delta c_t = \mu(s_t) + \sigma_c \epsilon_t, \quad \epsilon_t \sim N(0,1),$$

$$P(s_t = l | s_{t-1} = l) = p_{ll}, \; P(s_t = h | s_{t-1} = h) = p_{hh}$$

$$\Delta d_t = \lambda \Delta c_t + g_d + \tilde{\sigma}_d \epsilon_{d,t}, \; \epsilon_{d,t} \sim N(0,1)$$

- Mean state s_t: unobservable; belief: $\pi_t = \Pr(s_{t+1} = h | \mathcal{I}_t)$
- Bayes updating: given π_0

$$\pi_{t+1} = \frac{p_{hh} f(\Delta c_{t+1} | s_{t+1} = h) \pi_t + (1 - p_{ll}) f(\Delta c_{t+1} | s_{t+1} = l)(1 - \pi_t)}{f(\Delta c_{t+1} | s_{t+1} = h) \pi_t + f(\Delta c_{t+1} | s_{t+1} = l)(1 - \pi_t)}$$

$$f(\Delta c | s_t) \propto \exp\left[-\frac{(\Delta c - \mu(s_t))^2}{2\sigma_c^2}\right]$$

AAMS: Ju and Miao (2012)

- Value function

$$V(C;\pi_t) = \left[(1-\beta)C_t^{1-1/\psi} + \beta\left\{\mathcal{R}_t\left(V\left(C;\pi_{t+1}\right)\right)\right\}^{1-1/\psi}\right]^{\frac{1}{1-1/\psi}},$$

- Ambiguity-aversion-CE

$$\mathcal{R}_t\left(V\left(C;\pi_{t+1}\right)\right) = \left(\mathbb{E}_{\pi_t}\left[\left(\mathbb{E}_{\{s_{t+1},t\}}\left[V\left(C;\pi_{t+1}\right)^{1-\gamma}\right]\right)^{\frac{1-\eta}{1-\gamma}}\right]\right)^{\frac{1}{1-\eta}}$$

- γ: RRA, ψ: EIS
- $\eta > \gamma$: ambiguity aversion
- $\eta = \gamma$: ambiguity neutrality
- We do not consider $\eta < \gamma$ — might imply "ambiguity seeking"
- η: ambiguity aversion parameter

AAMS: Ju and Miao (2012)

Interpretation

- The compound distribution is not available
- Aversion to a mean-preserving-spread in the distribution of conditional certainty equivalent of future utility (conditional on s_{t+1})
- See Klibanoff, Marinacci and Mukerji (2005, 2009) and Hayashi and Miao (2011)

Model EZMS: suppressing ambiguity aversion

AAMS: Ju and Miao (2012)

- Stochastic discount factor

$$M_{t,t+1} = \underbrace{\beta \left(\frac{C_{t+1}}{C_t}\right)^{-1/\psi}}_{\text{short run risk}} \underbrace{\left(\frac{V_{t+1}}{\mathcal{R}_t\left(V_{t+1}\right)}\right)^{1/\psi-\gamma}}_{\text{long run risk}} \underbrace{\left(\frac{\left(\mathbb{E}_{s_{t+1},t}\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}}{\mathcal{R}_t\left(V_{t+1}\right)}\right)^{-(\eta-\gamma)}}_{\text{ambiguity aversion}}$$

- Risk-free rate $R_t^f = 1/\mathbb{E}_t\left[M_{t,t+1}\right]$
- Equity return $R_{t+1} = \dfrac{P_{t+1}+D_{t+1}}{P_t}$ satisfies Euler equation

$$\mathbb{E}_t\left[M_{t,t+1}R_{t+1}\right] = 1$$

or equivalently

$$\frac{P_t}{D_t} = \mathbb{E}_t\left[M_{t,t+1}\left(1+\frac{P_{t+1}}{D_{t+1}}\right)\frac{D_{t+1}}{D_t}\right]$$

AAMSSV

- Incorporating (*exogenous*) time-varying conditional volatility

$$\Delta c_t = \mu\left(s_t^\mu\right) + \sigma\left(s_t^\sigma\right)\epsilon_t, \quad \epsilon_t \sim N(0,1)$$

- transition probabilities

$$\begin{aligned} P\left(s_t^\mu = l | s_{t-1}^\mu = l\right) &= p_{ll}^\mu,\ P\left(s_t^\mu = h | s_{t-1}^\mu = h\right) = p_{hh}^\mu, \\ P\left(s_t^\sigma = l | s_{t-1}^\sigma = l\right) &= p_{ll}^\sigma,\ P\left(s_t^\sigma = h | s_{t-1}^\sigma = h\right) = p_{hh}^\sigma \end{aligned}$$

- s_t^μ: unobservable; s_t^σ: observable and independent of s_t^μ
- Bayes updating:

$$\pi_{t+1} = \frac{p_{hh}^\mu f\left(\Delta c_{t+1} | s_{t+1}^\mu = h, s_{t+1}^\sigma\right)\pi_t + \left(1 - p_{ll}^\mu\right) f\left(\Delta c_{t+1} | s_{t+1}^\mu = l, s_{t+1}^\sigma\right)(1-\pi_t)}{f\left(\Delta c_{t+1} | s_{t+1}^\mu = h, s_{t+1}^\sigma\right)\pi_t + f\left(\Delta c_{t+1} | s_{t+1}^\mu = l, s_{t+1}^\sigma\right)(1-\pi_t)}$$

where $f\left(\Delta c | s_{t+1}^\mu, s_{t+1}^\sigma\right)$ is conditional density

$$f\left(\Delta c | s_{t+1}^\mu, s_{t+1}^\sigma\right) \propto \frac{1}{\sigma\left(s_{t+1}^\sigma\right)} \exp\left[-\frac{\left(\Delta c - \mu\left(s_{t+1}^\mu\right)\right)^2}{2\sigma\left(s_{t+1}^\sigma\right)^2}\right]$$

AAMSSV

Value function

$$V\left(C;\pi_t,s_t^\sigma\right) = \left[(1-\beta)C_t^{1-1/\psi}+\beta\left\{\mathcal{R}_t\left(V\left(C;\pi_{t+1},s_{t+1}^\sigma\right)\right)\right\}^{1-1/\psi}\right]^{\frac{1}{1-1/\psi}},$$

$$\mathcal{R}_t\left(V\left(C;\pi_{t+1},s_{t+1}^\sigma\right)\right) = \left(\mathbb{E}_{\pi_t}\left[\left(\mathbb{E}_{\left\{s_{t+1}^\mu,s_t^\sigma,t\right\}}\left[V\left(C;\pi_{t+1},s_{t+1}^\sigma\right)^{1-\gamma}\right]\right)^{\frac{1-\eta}{1-\gamma}}\right]\right)^{\frac{1}{1-\eta}}$$

SDF

$$M_{t,t+1}=\beta\left(\frac{C_{t+1}}{C_t}\right)^{-1/\psi}\left(\frac{V_{t+1}}{\mathcal{R}_t\left(V_{t+1}\right)}\right)^{1/\psi-\gamma}\left(\frac{\left(\mathbb{E}_{\left\{s_{t+1}^\mu,s_t^\sigma,t\right\}}\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}}{\mathcal{R}_t\left(V_{t+1}\right)}\right)^{-(\eta-\gamma)}$$

AALRRSV

A version of the LRR model

$$\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_{t+1} + \sigma_t \epsilon_{c,t+1} \\
\Delta d_{t+1} &= \mu_d + \lambda x_{t+1} + \varphi_d \sigma_t \epsilon_{d,t+1} \\
x_{t+1} &= \rho_x x_t + \varphi_x \sigma_t \epsilon_{x,t+1} \\
\sigma^2_{t+1} &= \mu^2_\sigma + \rho_\sigma \left(\sigma^2_t - \mu^2_\sigma\right) + \sigma_w \epsilon_{w,t+1}
\end{aligned}$$

$\epsilon_{c,t+1}, \epsilon_{d,t+1}, \epsilon_{x,t+1}, \epsilon_{w,t+1} \sim i.i.d.N\,(0,1)$

- x_t: LRR component
- σ_t: time-varying economic uncertainty
- Assume the LRR component x_t is unobserved $\longrightarrow$ ambiguity about conditional mean growth state

AALRRSV

- The agent observes the realizations of Δc_{t+1} and Δd_{t+1} contemporaneously but never observes the realization of x_t or $(\epsilon_{c,t}, \epsilon_{d,t}, \epsilon_{x,t})$
- Suppose x_0 has a Gaussian prior
- Bayes updating: Kalman filter
  - Gaussian posterior: $x_{t+1} \sim N\left(\hat{x}_{t+1}, \nu_{t+1}\right)$
  - Updating laws of motion for $\hat{x}_{t+1}$ and ν_{t+1} are obtained by applying Kalman filter
- σ_t is assumed to be observable

AALRRSV

Value function $V_t = V_t(C; \hat{x}_t, \nu_t, \sigma_t)$

$$
\begin{aligned}
V_t &= \left[(1-\beta)C_t^{1-1/\psi} + \beta \left\{\mathcal{R}_t\left(V_{t+1}\right)\right\}^{1-1/\psi}\right]^{\frac{1}{1-1/\psi}}, \\
\mathcal{R}_t\left(V_{t+1}\right) &= \left(\mathbb{E}_{\{\hat{x}_t,\nu_t\}}\left[\left(\mathbb{E}_{\{x_t,\sigma_t,t\}}\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1-\eta}{1-\gamma}}\right]\right)^{\frac{1}{1-\eta}}.
\end{aligned}
$$

SDF

$$
M_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{V_{t+1}}{\mathcal{R}_t\left(V_{t+1}\right)}\right)^{\frac{1}{\psi}-\gamma}\left(\frac{\left(\mathbb{E}_{\{x_t,\sigma_t,t\}}\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}}{\mathcal{R}_t\left(V_{t+1}\right)}\right)^{-(\eta-\gamma)}
$$

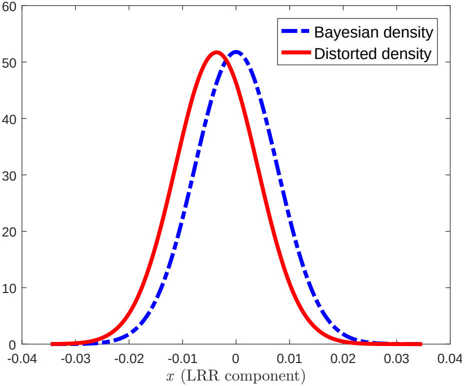
AALRRSV: an extension of Bansal and Yaron (2004)

- Bayesian density of x_t: $x_t \sim N\left(\hat{x}_t, \nu_t\right)$
- Distorted density of x_t:

$$\tilde{f}\left(x_t|\hat{x}_t,\nu_t,t\right) = \underbrace{\frac{\left(\mathbb{E}_t\left[V_{t+1}^{1-\gamma}|x_t\right]\right)^{-\frac{\eta-\gamma}{1-\gamma}}}{\int\left(\mathbb{E}_t\left[V_{t+1}^{1-\gamma}|x_t\right]\right)^{-\frac{\eta-\gamma}{1-\gamma}} f\left(x_t|\hat{x}_t,\nu_t\right)dx_t}}_{\text{ambiguity distortion}} \underbrace{f\left(x_t|\hat{x}_t,\nu_t\right)}_{\text{Bayesian}}$$

AALRRSV

Figure: **Model AALRRSV: Bayesian and distorted densities of x**



Notes: The distorted density is generated from the model solution. The state vector is assumed to take the value ($\hat{x}_t = 0$, $\nu_t = \bar{\nu}$ (steady-state) and $\sigma_t = \mu_\sigma$). Parameter values are set at mean values of Bayesian estimates.

EZLRRSV

- LRR model: Bansal, Kiku and Yaron (2012)

$$
\begin{aligned}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t \epsilon_{c,t+1} \\
\Delta d_{t+1} &= \mu_d + \lambda x_{t+1} + \varphi_d \sigma_t \epsilon_{d,t+1} + \varphi_c \sigma_t \epsilon_{c,t+1} \\
x_{t+1} &= \rho_x x_t + \varphi_x \sigma_t \epsilon_{x,t+1} \\
\sigma^2_{t+1} &= \mu^2_\sigma + \rho_\sigma \left(\sigma^2_t - \mu^2_\sigma\right) + \sigma_w \epsilon_{w,t+1}
\end{aligned}
$$

- Epstein-Zin's recursive utility

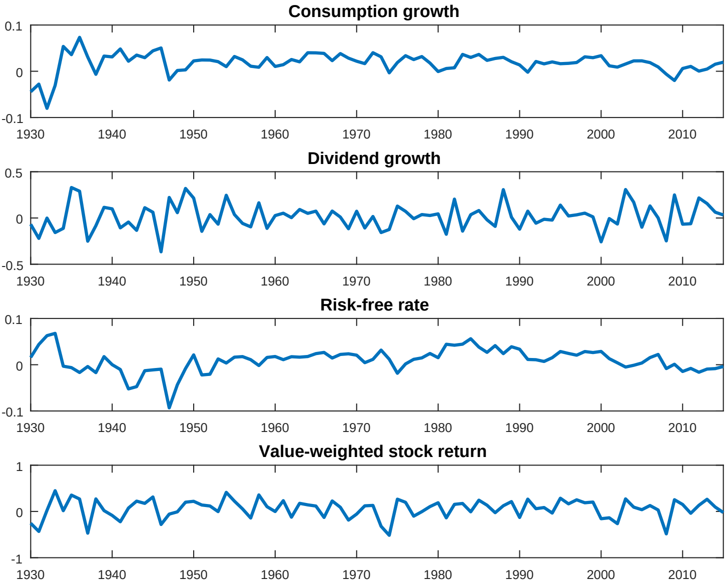
$$
V_t(C) = \left[(1-\beta) C_t^{1-1/\psi} + \beta \left\{ \mathbb{E}_t \left(V_{t+1}(C)^{1-\gamma} \right) \right\}^{1-1/\psi} \right]^{\frac{1}{1-1/\psi}},
$$

Model Summary

Model	State variables	Parameters
AAMS	π_t	$\{\beta, \gamma, \psi, \eta, \mu_h, \mu_l, p_{hh}, p_{ll}, \sigma, \lambda, \sigma_d\}$
AAMSSV	$(\pi_t,\ s_t^\sigma)$	$\{\beta, \gamma, \psi, \eta, \mu_h, \mu_l, p_{hh}^\mu, p_{ll}^\mu, \sigma_h, \sigma_l, p_{hh}^\sigma, p_{ll}^\sigma, \lambda, \sigma_d\}$
AALRRSV	$(\hat{x}_t,\ \nu_t,\ \sigma_t)$	$\{\beta, \gamma, \psi, \eta, \mu_c, \rho_x, \varphi_x, \lambda, \varphi_d, \mu_\sigma, \rho_\sigma, \sigma_w\}$
EZMS	π_t	$\{\beta, \gamma, \psi, \mu_h, \mu_l, p_{hh}, p_{ll}, \sigma, \lambda, \sigma_d\}$
EZLRRSV	(x_t,σ_t^2)	$\{\beta, \gamma, \psi, \mu_c, \rho_x, \varphi_x, \lambda, \varphi_d, \varphi_c, \mu_\sigma, \rho_\sigma, \sigma_w\}$

- Model solution: projection methods with Chebyshev polynomials
- Simulate consumption growth, dividend growth, risk-free rate and equity return $\left(\Delta c_t, \Delta d_t, r_t^f, r_t\right)$ from each model
- Bayesian MCMC: 60,000 simulations
- Loose priors
- Parallel computing on multi-cores processors

Data


Consumption growth
0.1
0
-0.1
1930
1940
1950
1960
1970
1980
1990
2000
2010
Dividend growth
0.5
0
-0.5
Risk-free rate
0.1
0
-0.1
Value-weighted stock return
1
0
-1

GSM Estimation Results: AAMS

	Posterior			
Param	Mean	Median	5%	95%
β	0.975	0.974	0.969	0.985
γ	2.841	3.063	0.563	4.563
ψ	2.040	2.031	1.781	2.406
η	6.959	6.938	5.063	8.938
p_{ll}	0.835	0.839	0.786	0.888
p_{hh}	0.996	0.997	0.994	0.997
μ_l	-0.039	-0.039	-0.048	-0.031
μ_h	0.022	0.022	0.016	0.029
λ	3.420	3.422	2.703	4.203
σ_c	0.019	0.019	0.015	0.022
σ_d	0.137	0.137	0.113	0.168
BIC	832.14			
Log lik.	-392.32			

GSM Estimation Results: AAMSSV

	Posterior			
Param	Mean	Median	5%	95%
β	0.982	0.984	0.972	0.991
γ	1.167	0.875	0.125	4.125
ψ	1.357	1.348	1.090	1.668
η	10.252	10.125	6.875	13.625
p^{μ}_{ll}	0.668	0.686	0.504	0.746
p^{μ}_{hh}	0.996	0.998	0.984	0.999
μ_l	-0.056	-0.057	-0.068	-0.042
μ_h	0.023	0.023	0.014	0.033
p^{σ}_{ll}	0.986	0.990	0.948	0.996
p^{σ}_{hh}	0.982	0.984	0.957	0.995
σ_l	0.013	0.012	0.004	0.022
σ_h	0.038	0.038	0.029	0.050
λ	2.739	2.641	1.953	4.016
σ_d	0.159	0.157	0.122	0.210
BIC	746.31			
Log lik.	-342.93			

GSM Estimation Results: AALRRSV

	Posterior			
Parameter	Mean	Median	5%	95%
β	0.986	0.987	0.979	0.992
γ	4.683	5.031	1.719	6.406
ψ	1.225	1.113	1.012	1.785
η	23.371	23.500	10.500	35.500
μ_c	0.019	0.019	0.017	0.020
ρ_x	0.941	0.941	0.926	0.957
ϕ_x	0.248	0.248	0.197	0.295
λ	3.555	3.453	2.953	4.672
ϕ_d	4.877	4.906	3.844	5.844
μ_s	0.020	0.020	0.019	0.021
ρ_s	0.950	0.950	0.950	0.950
σ_w	2.57E-04	2.54E-04	2.37E-04	2.79E-04
BIC	765.09			
Log lik.	-356.64			

GSM Estimation Results: EZLRRSV

	Posterior			
Param	Mean	Median	5%	95%
β	0.982	0.982	0.977	0.989
γ	8.431	8.531	6.219	10.438
ψ	1.732	1.758	1.227	2.117
μ_c	0.019	0.019	0.018	0.021
ρ_x	0.908	0.918	0.863	0.962
ϕ_x	0.189	0.184	0.145	0.245
λ	3.167	3.141	2.547	3.922
ϕ_d	4.602	4.594	3.891	5.344
ϕ_c	2.355	2.297	1.328	3.547
μ_s	0.021	0.021	0.020	0.022
ρ_s	0.950	0.950	0.950	0.950
σ_w	2.28E-04	2.31E-04	2.13E-04	2.46E-04
BIC		815.02		
Log lik.		-381.61		

GSM Estimation Results: EZLRRSV

- Log likelihood computation leads to the model ranking AAMSSV$\succ$AALRRSV$\succ$EZLRRSV$\succ$EZMS$\succ$AAMS
- BIC leads to the model ranking AAMSSV$\succ$AALRRSV$\succ$EZMS$\succ$EZLRRSV$\succ$AAMS

Asset Pricing Implications

	$\mathbb{E}(r_t^f)$	$\sigma(r_t^f)$	$\mathbb{E}(r_t - r_t^f)$	$\sigma(r_t - r_t^f)$	$\mathbb{E}(VRP_t)$	$\sigma(VRP_t)$	MPR
Data	1.41	2.82	5.32	17.77	11.07	24.94	N.A.
				AAMS			
Mean	1.595	1.541	5.812	18.441	12.955	9.300	1.280
Median	1.399	1.598	6.349	18.266	12.644	8.833	1.308
Std	0.800	0.286	1.738	2.220	2.967	2.720	0.341
95%	2.941	1.951	8.033	22.729	18.860	14.495	1.819
5%	0.444	1.951	3.010	15.378	8.332	5.411	0.758
				AAMSSV			
Mean	1.183	1.680	6.371	22.818	17.090	14.007	2.987
Median	1.267	1.673	5.910	22.579	14.678	10.430	2.790
Std	0.959	0.580	3.542	4.469	12.092	13.418	1.414
95%	2.465	2.685	14.446	30.242	35.407	39.337	5.970
5%	-0.622	0.800	1.650	15.633	4.835	3.878	1.205
				AALRRSV			
Mean	1.079	1.555	7.841	21.134	-0.744	1.752	1.312
Median	1.095	1.581	7.817	21.103	-0.288	1.086	1.142
Std	0.473	0.259	1.853	3.629	1.263	1.782	0.772
95%	1.818	1.946	10.838	28.345	0.406	6.490	2.976
5%	0.226	1.128	4.863	16.106	-3.218	0.438	0.489

Asset pricing implications

	$\mathbb{E}(r_t^f)$	$\sigma(r_t^f)$	$\mathbb{E}(r_t - r_t^f)$	$\sigma(r_t - r_t^f)$	$\mathbb{E}(VRP_t)$	$\sigma(VRP_t)$	MPR
Data	1.41	2.82	5.32	17.77	11.07	24.94	N.A.
				EZMS			
Mean	1.282	2.100	2.919	40.489	58.704	67.854	0.698
Median	1.278	2.035	2.727	41.471	54.164	65.354	0.647
Std	0.468	0.394	1.634	8.211	26.877	35.106	0.208
95%	2.107	2.902	6.007	52.464	107.602	128.264	1.128
5%	0.511	1.579	0.706	28.070	21.067	21.228	0.454
				EZLRRSV			
Mean	1.708	0.973	4.318	17.633	1.436	0.549	0.569
Median	1.686	0.856	4.458	17.560	1.398	0.524	0.573
Std	0.336	0.385	1.150	1.906	0.396	0.280	0.078
95%	2.289	1.589	5.999	20.903	2.137	1.130	0.692
5%	1.169	0.718	2.337	14.413	0.858	0.200	0.428

Conclusion

- Estimate several consumption-based asset pricing models with and without smooth ambiguity preferences.
    - Ju and Miao (2012), "AAMS"
    - An extension of Ju and Miao (2012) with time-varying volatility, "AAMSSV"
    - An extension of Bansal and Yaron (2004) LRR model with Bayesian filtering and smooth ambiguity, "AALRRSV"
    - Comparison to baseline models
        - Bansal, Kiku and Yaron (2012) LRR model, "EZLRRSV"
        - Regime-switching model under Epstein-Zin recursive utility, "EZMS"
- GSM Bayesian estimation coupled with SNP estimation of the auxiliary model

Conclusion

Findings

- Quantitative effects of smooth ambiguity on asset returns are significant
- The distinction between risk aversion and ambiguity aversion is robust to the intertemporal substitution effect
- The statistical support for smooth ambiguity is robust to specifications of consumption and dividend processes
- The model comparison shows that models with ambiguity, learning and time-varying volatility are preferred to the long-run risk model,
- High estimates of ψ: preference for early resolution of uncertainty
- Mean&vol. regimes in consumption growth; the LRR component